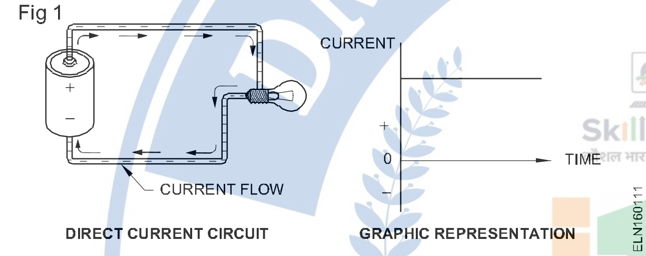
Alternating current - terms & definitions - vector diagrams

Objectives: At the end of this lesson you shall be able to
- **state the features of direct current**
- **list out the advantages of DC over AC**
- **compare the features of DC and AC**
- **explain the generation of alternating current and terms used**
- **state the advantages of AC over DC.**

Direct current (DC): Electric current can be defined as the flow of electrons in a circuit. Based on the electron theory, electrons flow from the negative (–) polarity to the positive (+) polarity of a voltage source.

Direct current (DC) is the current that flows only in one direction in a circuit. (Fig 1) The current in this type of circuit is supplied from a DC voltage source. Since the polarity of a DC source remains fixed, the current produced by it flows in one direction only.

Fig 1



Advantages of DC over AC

1 DC needs only two wires of transmission, while a 3 phase AC may need upto 4 wires.

2 The corona loss associated with DC is negligible while for AC it increases with its frequency.

3 The skin effect is also observed in AC leading to problems in transmission conductor designs.

4 No inductive and capacitive losses.

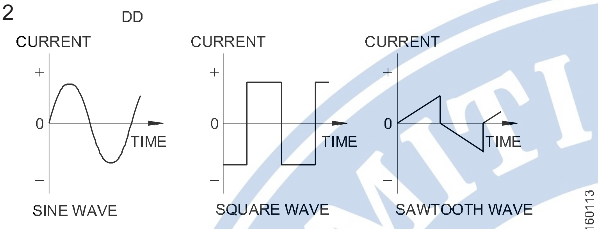
Comparison of AC and DC

	Alternating current	Direct current
Amount of energy that can be carried	Safe to transfer over longer city distances and can provide more power.	Voltage of DC cannot travel very far until it begins to lose energy.
Cause of the direction of flow of electrons	Rotating magnet along the wire.	Steady magnetism along the wire.
Frequency	The frequency of alternating current is 50Hz or 60Hz depending upon the country.	The frequency of direct current is zero.
Direction	It reverse its direction while flowing in a circuit.	It flows in one direction in the circuit.
Current	It is the current of magnitude varying with time.	It is the current of constant magnitude.
Flow of electrons	Electrons keep switching directions - forward and backward.	Electrons move steadily in one direction or 'forward'.
Obtained from	AC generator and mains.	Cell or battery.
Passive parameters	Impedence.	Resistance only.
Power factor	Lies between 0 to 1.	Nil
Types	Sinusoidal, trapezoidal, triangular, square	Pure

Alternating current (AC): An alternating current (AC) circuit is one in which the direction and amplitude of the current flow change at regular intervals. The current in this type of circuit is supplied from an AC voltage source. The polarity of an AC source changes at regular intervals resulting in a reversal of the circuit current flow.

Alternating current usually changes in both value and direction. The current increases from zero to some maximum value, and then drops back to zero as it flows in one direction. This same pattern is then repeated as it flows in the opposite direction. The wave-form or the exact manner in which the current increases and decreases is determined by the type of AC voltage source used. (Fig 2)

Fig 2



AC WAVE FORMS

Alternating current generation: Alternating current is used wherever a large amount of electrical power is required. Almost all of the electrical energy supplied for domestic and commercial purposes is alternating current.

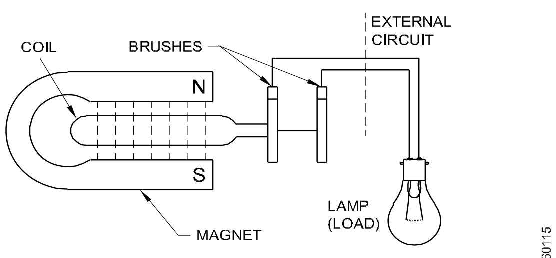
AC voltage is used because it is much easier and cheaper to generate, and when transmitted over long distances, the power loss is low.

Alternating current can be generated at higher voltages than DC. Some standard values of voltages are 1.1KV, 2.2.KV, 3.3KV for low capacity. The values are increased to 66 000, 110 000, 220 000, 400 000 volts for transmission over long distances. At the load area, the voltage is decreased to working values of 240V and 415V.

A generator is a machine that uses magnetism to convert mechanical energy into electrical energy. The generator principle, simply stated, is that a voltage is induced in a conductor whenever the conductor is moved through a magnetic field so as to cut the lines of magnetic force.

An AC generator produces an AC voltage by causing a loop of wire to turn within a magnetic field. This relative motion between the wire and the magnetic field causes a voltage to be induced between the ends of the wire. This voltage changes in magnitude and polarity as the loop is rotated within the magnetic field. (Fig 3)

Fig 3



SIMPLIFIED AC GENERATOR

The force required to turn the loop can be obtained from various sources. For example, very large AC generators are turned by steam turbines or by the movement of water.

The AC voltage induced in the armature coils is connected to a set of slip rings from which the external circuit receives the voltage through a set of brushes. An electromagnet is used to produce a stronger magnetic field.

The sine wave: The shape of the voltage wave-form generated by a coil rotating in a magnetic field is called a sine wave. The generated sine wave voltage varies in both voltage value and polarity.

If the coil is rotated at a constant speed, the number of magnetic lines of force cut per second varies with the position of the coil. When the coil is moving parallel to the magnetic field, it cuts no lines of force.

Therefore, no voltage is generated at this instant. When the coil is moving at right angles to the magnetic field, it cuts the maximum number of lines of force.

Therefore, maximum or peak voltage is generated at this instant. Between these two points the voltage varies in accordance with the sine of the angle at which the coil cuts the lines of force.

The coil is shown in five specific positions in Fig 4. These are intermediate positions which occur during one complete revolution of the coil position. The graph shows how the voltage increases and decreases in amount during one rotation of the loop.

Note that the direction of the voltage reverses each half-cycle. This is because, for each revolution of the coil, each side must first move down and then up through the field.

The sine wave is the most basic and widely used AC wave-form. The standard AC generator (alternator) produces a voltage of sine wave-form. Some of the important electrical characteristics and terms used when referring to AC sine wave voltage or current are as follows.

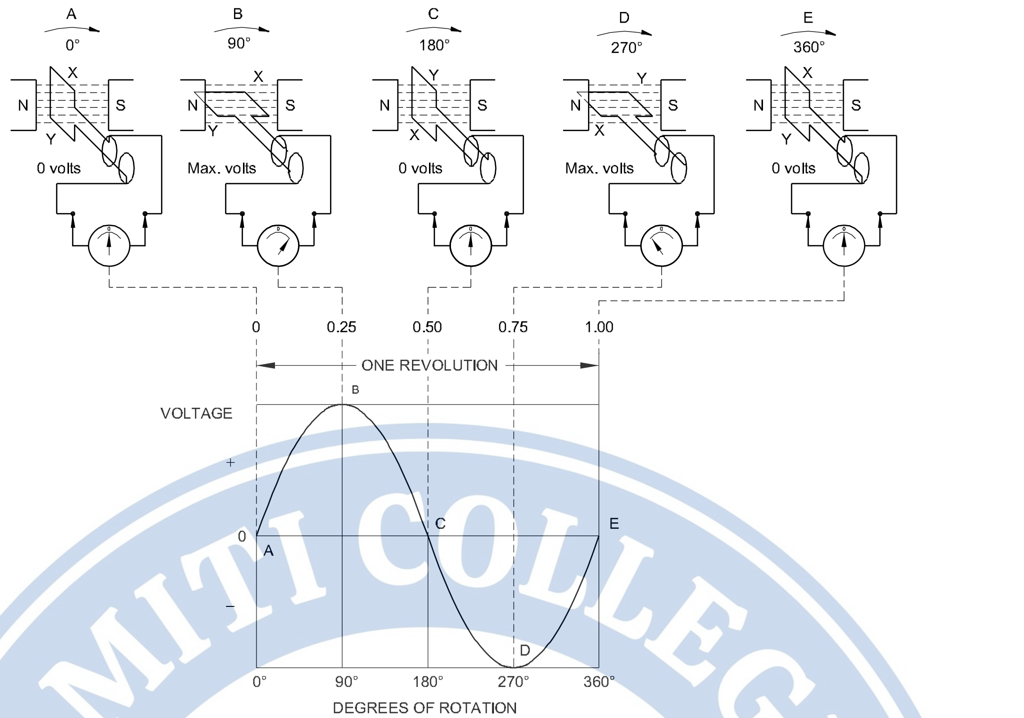
Cycle: One cycle is one complete wave of alternating voltage or current. During the generation of one cycle of output voltage, there are two changes in the polarity of the voltage.

These equal but opposite halves of a complete cycle are referred to as alternations. The terms positive and negative are used to distinguish one alternation from the other. (Fig 5)

Period: The time required to produce one complete cycle is called the period of the wave-form. In Fig 6, it takes 0.25 seconds to complete one cycle. Therefore, the period (T) of that wave-form is 0.25 seconds.

Frequency: The frequency of an AC sine wave is the number of cycles produced per second.(Fig 6) The unit of frequency is the hertz (Hz). For example, the 240V AC at your home has a frequency of 50 Hz.

Fig 4



GENERATION OF AN ALTERNATING VOLTAGE: AS THE LOOP ROTATES THROUGH THE MAGNETIC FIELD. THE AMOUNT AND POLARITY OF THE VOLTAGE CHANGES WITH ANGLE AND DIRECTION OF MOTION.

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Fig 5

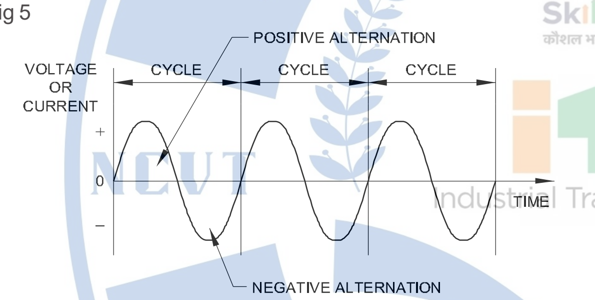

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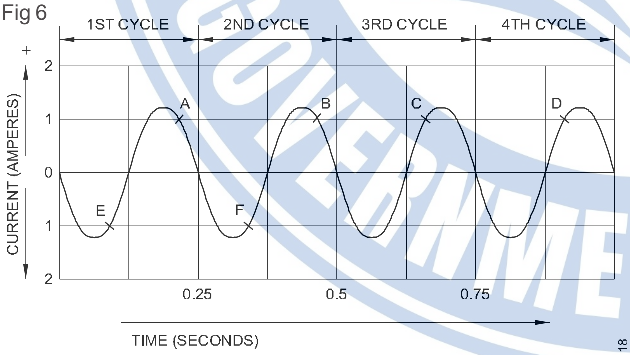
Fig 6



CYCLE, PERIOD, AND FREQUENCY. THE WAVEFORM HAS A PERIOD OF 0.25 SECONDS AND FOUR CYCLES PER SECOND.

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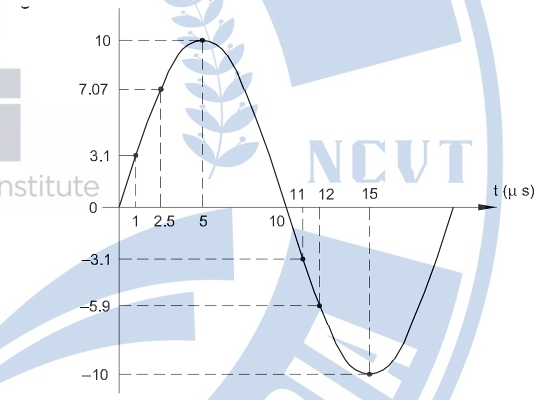
Fig 7



EXAMPLE OF INSANTANEOUS VALUES OF A SINE WAVE VOLTAGE

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Instantaneous value: The value of an alternating quantity at any particular instant is called instantaneous value. The instantaneous values of a sine wave voltage is shown in Fig 7. It is 3.1 volts at 1μs, 7.07 V at 2.5μs, 10V at 5μs, 0V at 10μs, - 3.1 volt at 11 μs and so on.

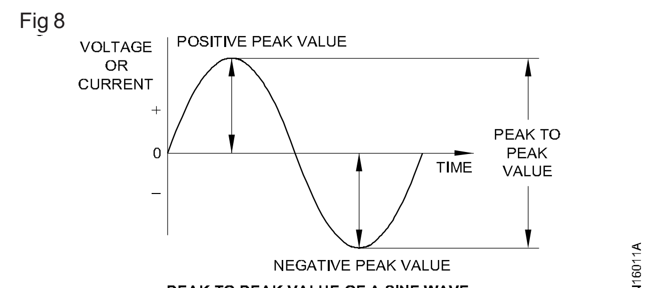
Peak value or maximum value: Each alternation of the sine wave is made up of a number of instantaneous values. These values are plotted at various heights above and below the horizontal line to form a continuous wave-form. (Fig 8)

The peak value of a sine wave refers to the maximum voltage or current value. Note that two equal peak values occur during one cycle.

Peak-to-peak value: The peak-to-peak value of a sine wave refers to its total overall value from one peak to the other. (Fig 8) It is equal to two times the peak value.

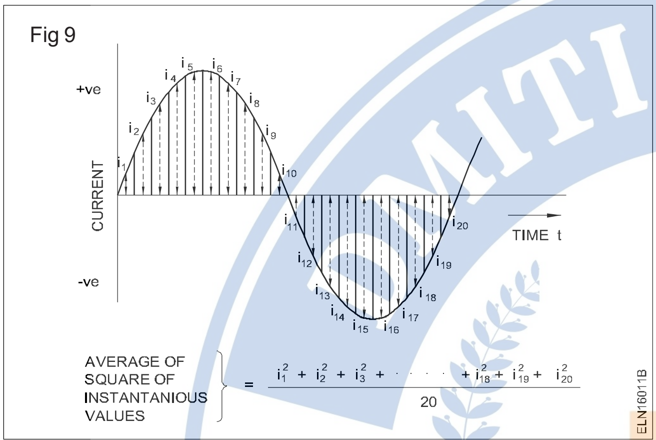
Effective value: The effective value of an alternating current is that value which will produce the same heating effect as a specific value of a steady direct current. In other words, an alternating current has an effective value of 1 ampere, if it produces heat at the same rate as the heat produced by 1 ampere of direct current, both flowing in the same value of resistance.

Another name for the effective value of an alternating current or voltage is the **root mean square (rms) value.** This term was derived from a method used to compute the value. The rms is calculated as follows.



PEAK TO PEAK VALUE OF A SINE WAVE

The instantaneous values for one cycle are selected for equal periods of time. Each value is squared, and the average of the squares is calculated (values are squared because the heating effect varies as square of the current or voltage). The square root of this is the rms value. (Fig 9)



By using this method it can be proved that the effective value of a sine wave of current is always equal to 0.707 times its peak value. A simple equation for calculating the effective value of sine wave is:

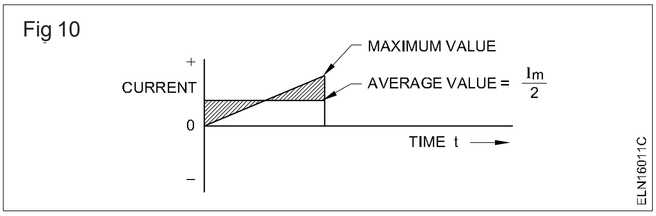
for voltage, $V = 0.707\ V_m$

for current, $I = 0.707\ I_m$

where subscript m refers to the maximum value.

When an alternating current or voltage is specified, it is always the effective value OR RMS valve that is meant, unless otherwise stated. Standard AC meters indicate effective values only.

Average value: It is sometimes useful to know the average value for one half cycle. If the current is changed at the same rate over the entire half cycle as in Fig 10, the average value would be one half of the maximum value.



It has been determined that the average value is equal to 0.637 times the maximum value for sine wave-form i.e.

for voltage, $V_{av} = 0.637\ V_m$

for current, $I_{av} = 0.637 I_m$

where subscript av refers to the average value and subscript m refers to the maximum value.

Form factor (k_f): Form factor is defined as the ratio of effective value to average value of half cycle.

For sinusoidal AC

$$k_f = \frac{0.707\ I_m}{0.6637\ I_m} = 1.11$$

where the subscript m refers to the maximum value.

Advantages of AC over DC:

1 AC voltages can be raised or lowered with ease. This makes it ideal for transmission purposes.

2 Large amounts of power can be transmitted at high voltage and low currents with minimum loss.

3 Because the current is low, smaller transmission wires can be used to reduce installation and maintenance costs.

4 AC is easy to generate than DC.

5 AC generators take higher efficiency than DC.

6 The loss of energy during transmission in negligible for AC in long distance.

7 The AC can be easily converted to DC.

8 It can easily stepup or stepdown using transformer.

Neutral and earth conductors

Objectives: At the end of this lesson you shall be able to
- **describe the purpose of earthing**
- **describe the two types of earthing**
- **differentiate between `neutral' and `earth wire'.**

Earthing: The importance of earthing lies in the fact that it deals with safety. One of the most important, but least understood, considerations in the design of electrical systems is that of earthing (grounding). The word `earthing' comes from the fact that the technique itself involves making a low-resistance connection to the earth or to the ground. The earth can be considered to be a large conductor which is at zero potential.

Purpose of earthing: The purpose of earthing is to provide protection to personnel, equipment and circuits by eliminating the possibility of dangerous or excessive voltage.

There are two distinct considerations in the earthing of an electrical system: earthing of one of the conductors of the wiring system, and earthing of all metal enclosures which contain electrical wires or equipment. The two types of earthing are:

- System earthing
- Equipment earthing.

System earthing: This consists of earthing one of the wires of the electrical system, such as the neutral, to limit the maximum voltage to earth under normal operating conditions.

Equipment earthing: This is a permanent and continuous bonding together (i.e. connecting together) of all non-current carrying metal parts of the electrical equipment to the system earthing electrode.

What is an earthing electrode?: A metal plate, pipe or other conductors electrically connected to the general mass of the earth is known as an earthing electrode. Earth electrodes shall be provided at generating stations, substations and consumer premises (in accordance with the requirements of IS : 3043-1966).

The neutral used in single phase system is to provide return path for load current to the source. Various method of neutral earthing is provided to serve neutral in single phase distribution at substation according to the requirements.

What is an `earth wire'?: A conductor connected to earth and usually situated in proximity to the associated line conductors which is used for equipment earthing is called an earth wire.

The purpose of equipment earthing: By connecting the metal work not intended to carry current to earth, a path is provided for leakage current which can be detected, and, if necessary, interrupted by the following devices.

– Fuses – Circuit breakers.

Use of vector diagram

Objective: At the end of this lesson you shall be able to
- **distinguish between scalar and vector quantity.**

Definition of scalar and vector quantity and phasor

Scalar quantity:A scalar quantity is a quantity which is determined by the magnitude alone, for example energy, volume, temperature etc.

Vector quantity: A vector quantity is a quantity which is represented by straight line with an arrow head to represent the magnitude and direction of it. For example, - force, velocity, weight.

Phasor: Phasor is a vector that is rotating at a constant angular velocity. A straight line with an arrow head is used to represent graphically the magnitude and phase of a sinusoidal alternating quantity (i.e. current, voltage and power) is called phasor.

Use of vector diagrams: The change which occurs in the value of an alternating voltage and/or current during a cycle can also be shown by using vector diagrams.

A vector is a line segment that has a define length and direction. A vector diagram is two or more vectors joined together to convey information. Vector diagrams drawn to scale can be used to determine instantaneous values of current and/or voltage.

Scalar quantity	Vector quantity
1. Scalar quantity can be presented by magnitude only, for example - energy, volume etc.	Vector quantity must represent magnitude and direction also, for example - force velocity etc.
2. Addition and substraction of scalar quantities can be done algebraically	Addition and subtraction of vector quantities cannot be done algebracially but by vector summation.

AC simple circuit

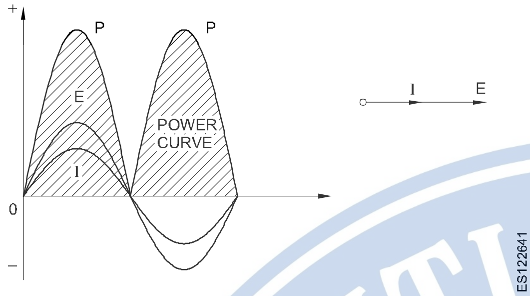
Objectives: At the end of this lesson you shall be able to
- **state phase relationship between voltage, current & power in pure resistance circuit**
- **state phase relationship between voltage, current & power in pure inductance circuit**
- **state phase relationship between voltage, current & power in pure capacitance circuit.**

Pure resistance circuit : A pure resistance circuit is one possessing neither inductance nor capacitance. Hence, if a current passes through the circuit. No back emf will be setup by any change in current. The applied voltage is required to overcome the ohmic drop only as in a dc circuit. So, we have, using effective values.

$$I = \frac{E}{R}$$

Since the current is proportional to the voltage, the wave form of current is exactly the same as that of voltage. When the voltage is zero the current is also zero. The two quantities are in phase with each other. Fig 1 shows a current wave, I, in phase with a voltage wave, E to obtain the power at each instant the current and voltage are multiplied together. With these products a new curve p, may be plotted. The power curve is positive during the first-half cycle because both the current and voltage are positive. During the second half-cycle both current and voltage are negative, hence their product will again be positive.

Fig 1

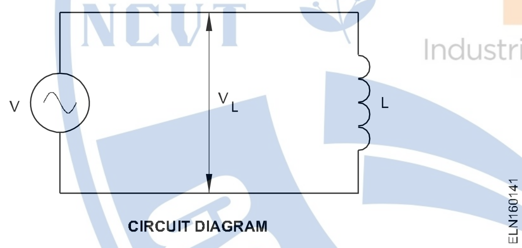


The power in a pure resistance circuit is given by the product of the effective voltage and current. Ie P = E.I.

Circuit with pure inductance only

A circuit with pure inductance alone can never be formed, because the source, the connecting wires, and the inductor all have some resistance. However, if these resistances are very small and have a much smaller effect on the circuit current than does the inductance, the circuit can be considered as containing only inductance. (Fig 2)

Fig 2



Phase difference: If two alternating quantities attain maximum value in the same direction after passing through zero value at different times, they are said to have a phase difference.

Phase difference can be expressed in fractions of a cycle. For more accuracy, phase difference is given in degrees. The terms `lead' and `lag' are used to describe the relative positions in time of two voltages or currents that are not in phase. The one that is ahead in time is said to lead, while the one behind lags. (Fig 3)

When maximum and minimum points of one voltage or current occur before the corresponding points of another voltage or current, the two are out of phase. When such a phase difference exists, one of the voltages or currents leads, and the other lags.

Phase relationship between current and voltage in a circuit with inductance only : When AC voltage is applied to an inductive circuit, the current lags behind the applied voltage by a quarter cycle or by 90°. (Fig 4)

Fig 3

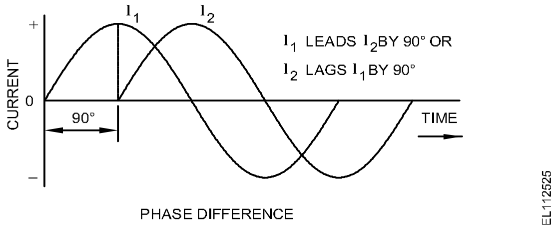
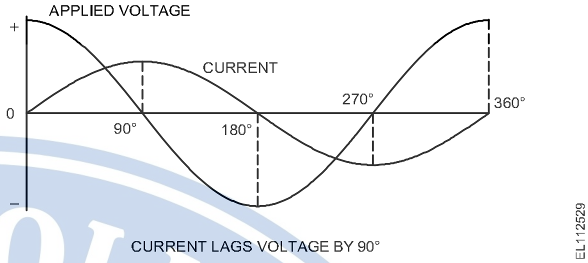


Fig 4



In a purely inductive circuit, the current lags behind the applied voltage by 90°. This is illustrated in the Fig 9 as wave-form. This also can be stated as voltage leads current. The vector diagram for both expressions is given in Figs 5 and 6.

Fig 5

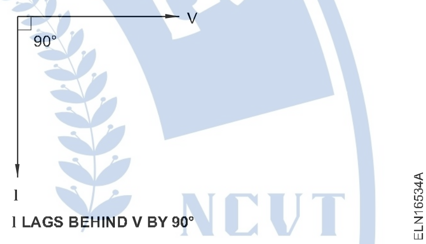
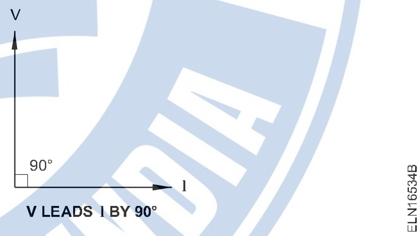


Fig 6



Inductive reactance: The cemf acts just like a resistance to limit the current flow. But cemf is discussed in terms of volts, so it cannot be used in Ohm's Law to compute the current. However, the effect of cemf can be given in terms of ohms. This effect is called inductive reactance, and is abbreviated as X_L. Since the cemf generated by an inductor is determined by the inductance (L) of the inductor, and the frequency (f) of the current, the inductive reactance must also depend on these things. The inductive reactance can be calculated by the equation

$$X_L = 2\pi fL$$

where X_L is the inductive reactance in ohms; f is the frequency of the current in cycles per second; and L is the inductance in henrys. The quantity 2π together actually represents the rate of change of the current, usually denoted by the Greek letter `ω' (Omega).

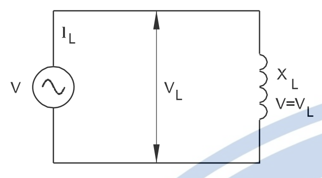
Since $2\pi = 2(3.14) = 6.28$, the Eqn. becomes similarly

$$L = \frac{X_L}{6.28\,f}$$

$$f = \frac{X_L}{6.28\,L}$$

In a circuit containing only inductance, Ohm's Law can be used to find the current and voltage by substituting X_L for R. (Fig 7)

Fig 7



CIRCUIT WITH ONLY X_L

$$I_L = \frac{V_L}{X_L}$$

$$X_L = \frac{V_L}{I_L}$$

$$V_L = I_L X_L$$

where I_L = current through the inductance, in amperes

V_L = voltage across the inductance, in volts

. X_L = inductive reactance in ohms

Pure capacitance circuit

Fig 8 shows an alternating emf E applied to the plates of a capacitor. When the voltage starts from zero value at 0.

Fig 8

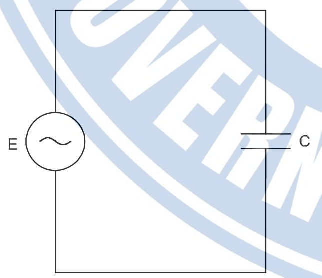


Fig 9 and increases positively, current flows into the capacitor and this current is also positive. As long as the emf across the capacitor plates increases, current flows into the capacitor.

When the instant L is reached, the increase of emf stops and current decreases to zero. Between L and M the emf decreases and current flows out of the capacitor so the capacitor discharges and as the current reverses its direction, the sign of the current become negative. This reversal of current is shown by the current wave I in Fig 5 after the voltage wave E goes through zero at M the emf is negative and the charge in the capacitor is reversed, so, the current the remains in the negative direction. This continues until the emf reaches its maximum value in the negative direction. At the instant N, the current reverse and again becomes positive charging and discharging of capacitor continue as long as the alternating emf is present across its plate.

Fig 9 shows that the alternating emf applied to a capacitor causes the current in the capacitor to lead the applied emf by 90°. This is shown by phasors in Fig 10.

Fig 9

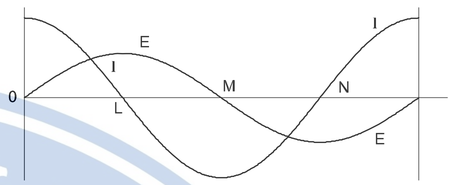
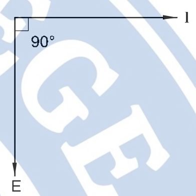


Fig 10



Capacitive reactance: The opposition offered to the flow of current by a capacitor is called capacitive reactance and is abbreviated Xc. Capacitive reactance can be calculated by:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

Where 2π is approximately 6.28

F is the frequency in Hz

C is the capacitance is farad and $\omega = 2\pi f$

Like its inductive counterpart - inductive reactance, capacitive reactance is expressed in ohms. Ohm's law can be also be applied to a circuit containing capacitive reactance only.

Example 1

A 10 µF capacitor is connected across a 250 V, 50 Hz supply. Calculate (a) the resistance of the capacitor and (b) the current.

Solution:

Reactance

$$X_C = \frac{1}{2\pi f C}$$

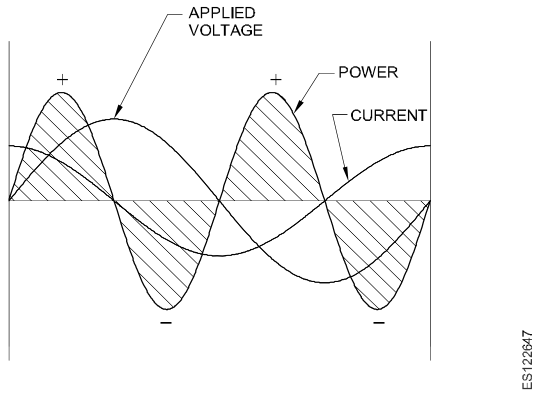
$$= \frac{1}{2 \times 3.14 \times 50 \times 10 \times 10^{-6}}$$

$$\text{Current} = \frac{250}{318.3} = 0.785A$$

The average power in a circuit containing only a capacitance is zero. This may be shown by plotting the power curve from the current and voltage curves (Fig 11) as was done for a circuit with the inductance only.

Fig 11 power curve for a purely capacitive circuit.

Fig 11



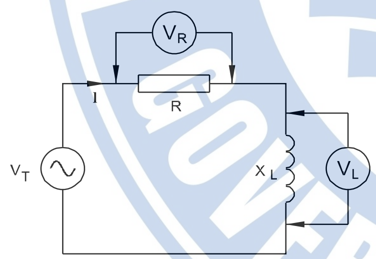
A.C. circuit with R & L in series

Objectives: At the end of this lesson you shall be able to
- **state the voltage and current relationship**
- **determine impedance of a series circuit with RL in series**
- **calculate power in a series circuit (with RL in series)**
- **calculate the power factor in RL series circuit.**

When resistance and inductance are connected in series, or in the case of a coil with resistance, the rms current I_L is limited by both X_L, and R however the current I is the same in X_L and R since they are in series, the voltage drop across R is V_R = IR and the voltage drop across X_L is $V_L = IX_L$. The current I through X_L must lag V_L by 90° because this is the phase angle between current through an inductance and its self-induced voltage. The current I through R, and its IR voltage drop, are in phase and so the phase angle is 0°.

Now let us apply the principle of phasor representation to a series circuit containing pure resistance and pure inductance. (Fig 1)

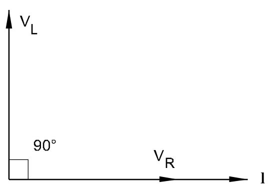
Fig 1



CIRCUIT WITH R & L IN SERIES

Since we are considering a series circuit, it is convenient if we draw the current phasor in the horizontal reference position because it is `common' to both the resistor and inductor. Superimposed upon this phasor is the voltage phasor across the resistor V_R. This is because the current and voltage are always in phase with each other in a pure resistor. (Fig 2)

Fig 2



VECTOR DIAGRAM OF R & L SERIES CIRCUIT

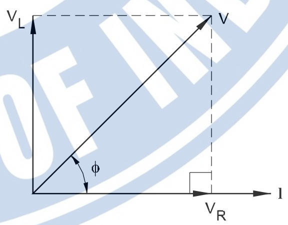
Similarly, the voltage phasor across the inductor V_L is drawn 90° ahead of the current I in other words leading the current phasor. This is because, as we know, the current always lags the inductor voltage by 90° in a pure inductance.

However, these two voltages are 90° out of phase with each other. This means that the total voltage across the series combination cannot be obtained simply by adding V_R to V_L algebraically. We must take into account the angle between them.

The applied voltage V is the (phasor) sum of V_R and V_L with the phase angle added.

This phasor addition can be carried out simply by constructing a parallelogram (a square in this case) and drawing the diagonal. This is shown in Fig 3. Clearly, the phasor sum V is less than the algebraic sum of V_L and V_R. Also, because V is the hypotenuse of a right-angled triangle, V is given by

Fig 3



PHASOR DIAGRAM OF R & L SERIES CIRCUIT

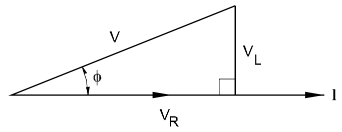
$$V^2 = V_R^2 + V_L^2$$

Impedance of a series RL circuit: The total opposition to current in a series, RL circuit, is called the **impedance** Z. It is the ratio of the total applied voltage V to the current I. Impedance is measured in ohms as are resistance and inductive reactance. But, as shown by the following, impedance is the vector sum of resistance and reactance.

Consider the `voltage triangle' for a series, RL circuit, as shown in Fig 4.

Given $V^2 = V_R^2 + V_L^2$ and V_R = IR and $V_L = IX_L$

Fig 4



(a) VOLTAGE TRIANGLE

then $V = \sqrt{(IR)^2 + (IX_L)^2}$

$= \sqrt{I^2R^2 + (I^2X_L)^2}$

$= \sqrt{I^2(R^2 + X_L{}^2)}$

$= I\sqrt{R^2 + X_L{}^2}$ and $\frac{V}{I} = \sqrt{R^2 + X_L{}^2}$

But $\frac{V}{I}$ is the impedance Z.

Therefore, $Z = \sqrt{R^2 + X_L{}^2}$ ohms

where Z is the impedance in ohms

R is the resistance in ohms

X_L is the inductive reactance in ohms

and $I = \frac{V}{Z}$ amperes (A).

Power factor: The ratio of the true power delivered to an AC circuit compared to the apparent power that the source must supply is called the power factor of the load.

If we examine any power triangle, we see that the ratio of the true power to the apparent power is cosine of the angle Ø.

Power factor $= \frac{W}{VA} = \cos\phi$

power factor must also be equal to $\frac{V_R}{V}$ and to $\frac{R}{Z}$

Power factor (PF) $= \frac{W}{VA} = \frac{V_R}{V} = \frac{R}{Z}$

What should be the power factor for a circuit containing pure resistance only?. As the phase angle Ø between current and voltages is $\phi = 0$.

$\cos\phi = 1$ and PF = 1.

Similarly the power factor for circuit containing pure inductance or pure capacitance only is zero as

$\cos\phi = \cos 90° =$ zero.

Example: An inductive circuit has a resistance of 2 ohms in series with an inductance of 0.015 henry. Find (i) current and (ii) power factor when connected across 200 volt 50 cycles per second supply mains.

Solution

$X_L = 2\pi fL = 2 \times 3.142 \times 50 \times 0.015 = 4.71$ ohms

$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(2)^2 + (4.71)^2}$

$= \sqrt{4 + 17.39} = \sqrt{26.19}$

i $I = \frac{200}{5.11} = 39.13$ amps

ii Power factor $= \frac{R}{Z} = \frac{2}{5.11} = 0.39$

Power and power factor in AC single phase circuit

Objective: At the end of this lesson you shall be able to

- **calculate power and power factor of a single phase AC circuit from the given relevant values.**

Power in pure resistance circuit: Power can be calculated by using the following formulae.

1) $P = V_R \times I_R$ watts

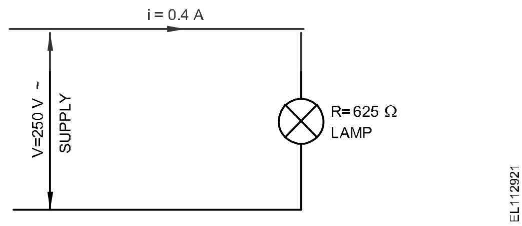
2) $P = I^2_R\ R$ watts

3) $P = \frac{E^2}{R}$ watts

Example 1: Calculate the power taken by an incandescent lamp rated 250V when it carries a current of 0.4A if the resistance is 625 ohms.(Fig 1)

$P = V_R \times I_R$

Fig 1



= 250 x 0.4

= 100 watts.

Alternately

$$P = I^2R$$

$$= 0.4 \times 0.4 \times 625$$

$$= 100 \text{ watts}$$

$$\text{or } P = \frac{E^2}{R} = \frac{250^2}{625}$$

$$P = \frac{250 \times 250}{625}$$

$$= 100 \text{ watts.}$$

Since the current and voltage are in phase, the phase angle is zero and the power factor is unity. Therefore, the power can be calculated with voltage and current itself.

Power in pure inductance: If an AC circuit contains only inductance, the voltage and current are 90° out of phase, and the circuit of the instantaneous values of voltage and current gives with positive and negative power. Net result is the power consumed in a pure inductive circuit is zero.

Power in pure capacitance: If an AC circuit contains only capacitor, the voltage and current are 90°. Out of phase and the product of instantaneous values of voltage and current gives both positive and negative power. Net result is the power consumed in a pure capacitive circuit is zero.

Most industrial installations have a lagging PF because of the large number of AC induction motors that are inherently inductive.

Effect of a low power factor

For a given quantity of true power if the power factor of the load is less than unity it requires a higher current to deliver. This higher current means that more energy is wasted in the feeder wires serving the motor. In fact, if an industrial installation has a power factor less than 85% (0.85) overall, a `power factor penalty' is assessed by the electric utility company. It is for this reason that power factor correction is necessary in large installations.

Power factor correction: In order to make the most efficient use of the current delivered to a load we desire a high PF or a PF that approaches unity.

A low PF is generally due to the large induction loads such as discharge lamps, induction motors, transformers etc. which take a lagging current and produce heat which returns to the generating station without doing any useful work as such it is essential to improve or correct the low PF so as to bring the current as closely in phase with the voltage as possible. That is the phase angle θ is made as small as possible. This is usually done by placing a capacitor load which produces a leading current.

The capacitor is to be connected in parallel with the inductive load.

R - C Series circuit

Objectives: At the end of this lesson you shall be able to
- **state the effect of frequency on capacitive reactance in R-C series circuit**
- **calculate power factor**
- **determine the power factor and phase angle**
- **state the R-C time constant while charging and discharging.**

In a circuit with capacitance, when the supply frequency (f) increses the capacitive reactance (X_C) decreases

$$X_C \propto \frac{1}{f}$$

When the capacitive reactance X_C increases the circuit current decreases.

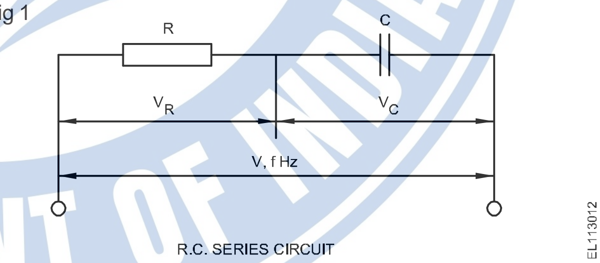
$$I \propto \frac{1}{X_C}$$

Therefore the increase in frequency (f) results in the increase of the circuit current in the capacitive circuit. When resistance (R), capacitance (C) and frequency f are known in a circuit, the power factor cos θ can be determined as follows. (Fig 1)

$$X_C = \frac{1}{2\pi fC}$$

$$Z = \sqrt{R^2 + X_C^2}$$

Fig 1



R.C. SERIES CIRCUIT

$$\text{Power factor, } \cos\theta = \frac{R}{Z}$$

Capacitive reactance X_C in a capacitive circuit can be determined with the formula

$$X_C = \frac{1}{2\pi fC}$$

where X_C = capacitive reactance in ohm

f = frequency in Hz

C = Capacitance in farad

Power consumed in a R-C series circuit can be determined using the formula

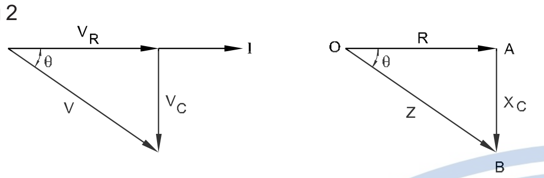
$P = VI \cos\theta$ where P = power in watts

I = current in ampere

$\cos\theta$ = power factor.

Vector diagram of voltages and their use to determine pf angle θ. (Fig 2)

Fig 2



a) VOLTAGE TRIANGLE   b) IMPEDENCE TRIANGLE

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$V_R = I_R$ drop across R (in phase with I)

$V_C = IX_C$ drop across capacitor (lagging I by 90°)

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

$\therefore Z = \sqrt{R^2 + X_C^2}$ where Z is the impedance of the circuit.

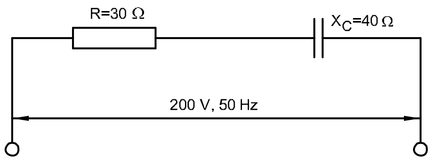
Power factor, $\cos\theta$ = R/Z.

From pf $\cos\theta$ the angle θ can be known referring to the Trignometric table.

Example 2: In RC series circuit shown in the diagram (Fig 3) obtain the following.

- Impedance in ohms

Fig 3



R.C. SERIES CIRCUIT

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- Current in amps
- True power in watts
- Reactive power in var
- Apparent power in volt amp.
- Power factor

Solution

1 Impedence (Z)

$$= \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50\Omega$$

2 Current $I = \frac{V}{Z} = \frac{200}{50} = 4A$

3 True power $W = I^2R = 4^2 \times 30 = 480W$
(Power consumed by capacitoir = zero)
$V_C = IX_C = 4 \times 40 = 160$ V

4 Reactive power VAR = $V_C I$ = 160 x 4 = 640 VAR

5 Apparent power VI = 200 x 4 = 800 VA

6 PF = $\cos\theta = \frac{R}{Z} = \frac{30}{50} = 0.6$

R.L.C Series circuit

Objectives: At the end of this lesson you shall be able to
- **draw the vector diagram of the voltage**
- **determine impedance**
- **solve problem.**

Resistance, Inductance and capacitance in series (Fig 1a) shows resistance R, inductive reactance XL and capacitive reactance Xc, are connected in series. The voltage across the circuit is E, the frequency is f and the current is I.

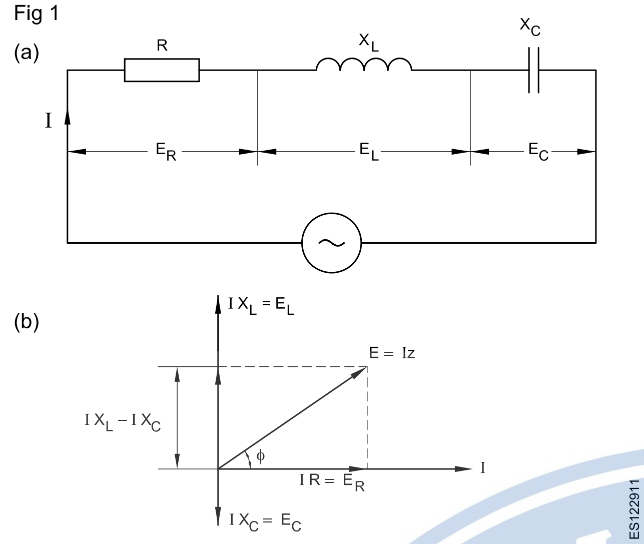
As this is a series circuit, the current is the same in all parts of the circuit, and for convenience the current phasor I is laid off horizontally in the circuit phasor diagram. The voltage E_R - IR across the resistance is in phase with the current and drawn to scale along the current phasor. The voltage E_L - IX_L across the inductance is drawn at right angles to the current and leading. The voltage $E_c = IX_c$ across the capacitor is drawn at right angles to the current and lagging.

The voltage across the inductance and that across the capacitance are in opposition Fig 1 (b) so that the resultant voltage of these two is their arithmetical difference. In Fig (1b) IX_L is shown greater than IX_c therefore, is subtracted directly form IX_L. The line voltage must be phasor sum of three voltages and is the hypotenuse of a right - angled triangle and is the hypotenuse of a right - angled triangle of which IR and IX_L - IX_c are the sides. Therefore,

$$E = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

Fig 1



= IZ

$$\therefore Z = \sqrt{(R)^2 + \left(X_L - X_C\right)^2}$$

And $I = \frac{E}{Z}$

The phase angle is found by

$$\tan\phi \frac{X_L - X_C}{R}$$

Example: A series circuit consist of a resistance of 20 ohms. An inductance of 0.2 Henry and a capacitance of 100 MFD is connected to 220 volts 50 HZ supply. Calculate

a the impedance of the circuit

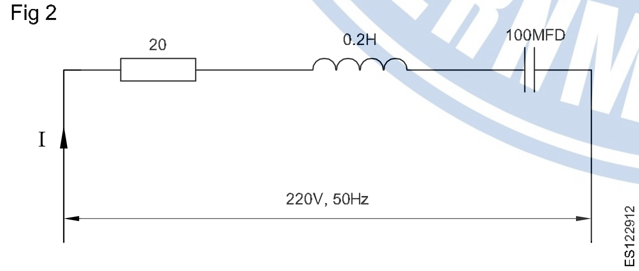
b the current flowing in the circuit

c power factor of the circuit

d power consumed in the circuit

e voltage drop in each element (Fig 2)

Fig 2



Solution:

R = 20 ohms

L = 0.2 Henry

C = 100 MFD

V = 220V

F = 50 Hz

Inductive reactance XL = 2π x 50 x 0.2 = 62.8 ohms

Capacitance reactance Xc.

$$= \frac{1}{2\pi fC} = \frac{10}{2\pi \times 50 \times 100} = 32\text{ohms}$$

a impedance $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$

$$= \sqrt{20^2 + (62.8 - 32)^2} = 36.7 \text{ ohms}$$

b current in the circuit I = V/Z = 220/36.7 = 5.99 amps

c Power factor = cos = R/Z = 20/36.7 = 0.54 (lag)

d Power P = VI Cos = 220 x 5.99 x 0.54 watts

P = 711.61 watts

E Voltage drop in R = IR = 5.99 x 20 = 119.8V

Voltage drop in L = IXL = 5.99 x 62.8 = 376.17V

Voltage drop in C = IXC = 5.99 x 32 = 191.68V.

Resonance circuit: When the value of X_L and X_C are equal, the voltage drop across them will be equal and hence they cancel each other. The value of voltage drops V_L and V_C may be much higher than the applied voltage. The impedance of the circuit will be equal to the resistance value. Full value of applied voltage appears across R and the current in the circuit is limited by the value of resistance only. Such circuits are used in electronic circuits like radio/TV turning circuits. When $X_L = X_C$ the circuit is said to be in resonance. As current will be maximum in series resonant circuits it is also called acceptor circuits. For a known value of L and C the frequency at which this occurs is called as resonant frequency. This value can be calculated as follows when $X_C = X_L$.

$$2\pi fL = \frac{1}{2\pi fC}$$

Hence resonance frequency fr = $\frac{1}{2\pi\sqrt{LC}}$

Note: Power factor angle is commonly denoted by Theta. In some pages of this text it is denoted by Phi. As such these terms are used alternatively in this text.

Series resonance circuit

Objectives: At the end of this lesson you shall be able to
- **explain the impedance of series resonance circuit**
- **state the condition for series resonance and its expression**
- **state the resonance frequency and its formula.**

Series resonance circuit

Impedance of series resonance circuit

A simple series LC circuit shown in Fig 1. In this series LC circuit,

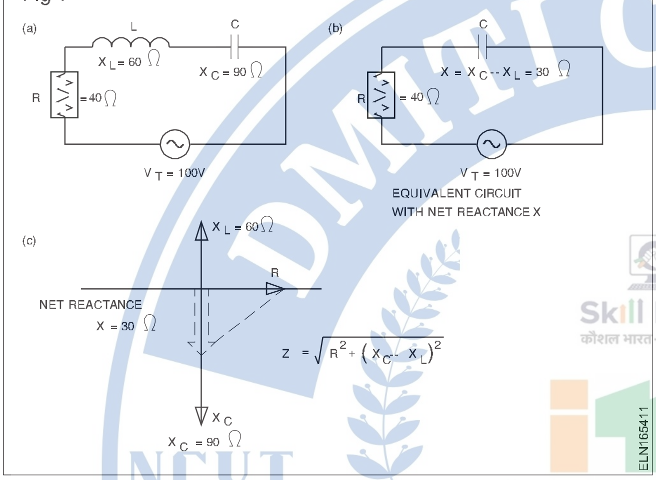


Fig 1

- resistance R is the total resistance of the series circuit(internal resistance) in ohms,
- X_L is the inductive reactance in ohms, and
- X_C is the total capacitive reactance in ohms.

In the circuit at Fig 1a, since the capacitive reactance(90Ω) is larger than inductive reactance(60Ω), the net reactance of the circuit will be capacitive. This is shown in Fig 1b.

> **Note: If the capacitive reactance was smaller than inductive reactance the net reactance of the circuit would have been inductive.**

All though the unit of measure of reactance and resistance is the same(ohms), the impedance, Z of the circuit is not given by the simple addition of R, X_L and X_C. This is because, X_L is +90° out of phase with R and X_C is -90° out of phase with R.

Hence the impedance Z of the circuit is the phasor addition of the resistive and reactive components as shown by dotted lines in Fig 1c. Therefore, Impedance Z of the circuit is given by,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

If X_L were greater than X_C, then the absolute value of impedance Z is will be,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For the circuit in Fig 2(a), total impedance Z is,

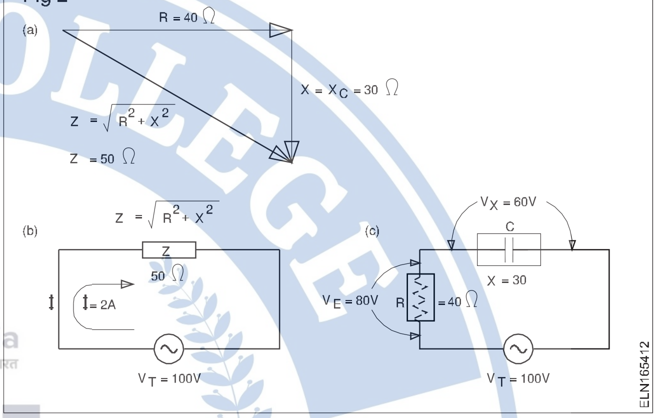


Fig 2

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{40^2 + 30^2}$$

$Z = 50\Omega$, Capacitive (because $X_C > X_L$)

Current I through the circuit is given by,

$$I = \frac{V}{Z} = \frac{100}{50\Omega} = 2 \text{ Amps}$$

Therefore, the voltage drop across the components will be,

V_R = voltage drop across R = I.R = 2x40 = 80 volts

V_L = voltage drop across L = I.X_L = 2x60 = 120 volts

V_C = voltage drop across C = I.X_C = 2x90 = 180 volts.

Since V_L and V_C are of opposite polarity, the net reactive voltage V_X is = 180 - 120 = 60V as shown in Fig 2.

Note that the applied voltage is not equal to the sum of voltage drops across reactive component X and resistive component. This is again because the voltage drops are not in phase. But the phasor sum of V_R and V_X will be equal to the applied voltage as given below,

$$V_T = \sqrt{V_R^2 + V_X^2}$$

$$= \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{80^2 + 60^2} = 100 \text{ volts(applied voltage).}$$

Phase angle θ of the circuit is given by,

$$\theta = \tan^{-1}\frac{X_C - X_L}{R}$$

Condition at which current through the RLC Series circuit is maximum

From the formula,

$Z = \sqrt{R^2 + (X_C - X_L)^2}$ it is clear that the total impedance Z of the circuit will become purely resistive when,

reactance $X_L = X_C$

In this condition, the impedance Z of the circuit will not only be purely resistive but also minimum.

Since the reactance of L and C are frequency dependent, at some particular frequency say f_r, the inductive reactance X_L becomes equal to the capacitive reactance X_C. In such a case, since the impedance of the circuit will be purely resistive and minimum, current through the circuit will be maximum and will be equal to the applied voltage divided by the resistance R.

Series resonance

From the above discussions it is found that in a series RLC circuit,

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\text{Current } I = \frac{V}{Z}$$

and,

$$\text{Phase angle } \theta = \tan^{-1}\frac{X_L - X_C}{R}$$

If the frequency of the signal fed to such a series LC circuit is increased from 0 Hz, as the frequency is increased, the inductive reactance($X_L = 2\pi fL$) increases linearly and the capacitive reactance ($X_C = 1/2\pi fL$) decreases exponentially.

At a particular frequency called the resonance frequency, f_r, the sum of X_L and X_C becomes zero($X_L - X_C = 0$).

From above, at resonant frequency ,

- Net reactance, X = 0 (i.e, $X_L = X_C$)
- Impedance of the circuit is minimum, purely resistive and is equal to R
- Current I through the circuit is maximum and equal to V/R
- Circuit current, I is in-phase with the applied voltage V (i.e. Phase angle = 0).

At this particular frequency f_r called resonance frequency, the series RLC is said to in a condition of series resonance.

Resonance occurs at that frequency when,

$X_L = X_C$ or $2\pi fL = 1/2\pi fC$

Therefore, **Resonance frequency, f_r** is given by,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \qquad \text{....[1]}$$

R-L, R-C and R-L-C parallel circuits

Objectives: At the end of this lesson you shall be able to
- **explain the admittance triangle and the relationship between conductance, susceptance and admittance**
- **explain susceptance, conductance and admittance by symbols.**

R-L Parallel circuit : When a number of impedances are connected in parallel across an AC voltage, the total current taken by the circuit is the phasor sum of the branch currents (Fig 1).

There are two methods for finding the total current.

- Admittance method
- Phasor method

Admittance method

The current in any branch $I = \frac{E}{Z}$

$$= E \times \left|\frac{1}{Z}\right| \text{ where } \left|\frac{1}{Z}\right|$$

is called the **admittance** of the circuit i.e. admittance is the reciprocal of impedance. Admittance is denoted by 'Y' (Fig 2).

$$I = E \times \left|\frac{1}{Z}\right| = EY \quad \text{or} \quad Y = \frac{I}{E}$$

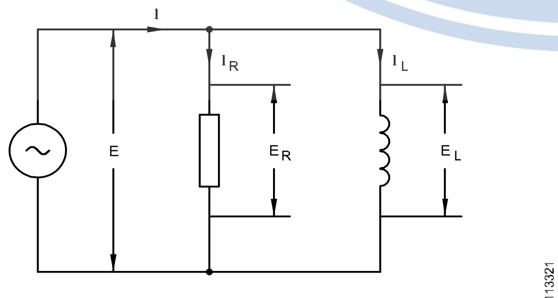
$$\therefore \text{Total admittance } (Y_T) = \frac{\text{total current}}{\text{common applied voltage}}$$

$$= \frac{\text{phasor sum of branch currents}}{\text{common applied voltage}}$$

= phase sum of separate admittance

Note: Supply voltage is referred as V or E interchangeably.

Fig 1

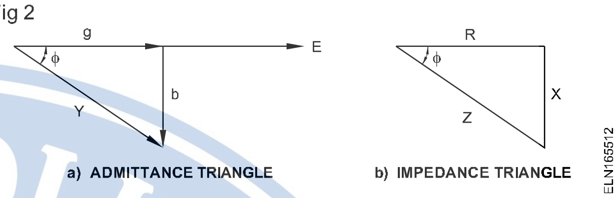


R.L. PARALLEL CIRCUIT

An admittance may be resolved into two components.

- A component in phase with the applied voltage called the conductance denoted by g.
- A component in quadrature (at right angle) with the applied voltage called **susceptance,** denoted by b.

Fig 2



$$g = Y \cos\phi = \frac{1}{Z} \times \frac{R}{Z}$$

$$= \frac{R}{Z^2} = \frac{R}{R^2 + Z^2}$$

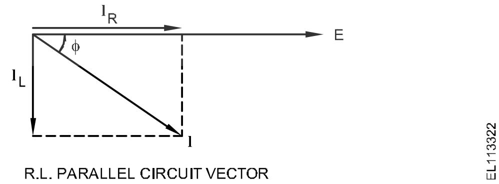
$$b = Y \sin\phi = \frac{1}{Z} \times \frac{X}{Z} = \frac{X}{Z^2}$$

$$= \frac{X}{R^2 + X^2}$$

The unit of admittance, conductance and susceptance is called the mho symbol ℧.

Relationship between branch current and supply voltage : In a R-L parallel circuit, the voltage across resistor (E_R) and inductor (E_L) are the same and equal to the supply voltage E. Hence E is the reference vector. The current through resistor (I_R) in phase with E_R is E. (Fig 3) The current through inductor (I_L) is lagging the E_L is E by 90^o. In short the current through resistor I_R is in phase and the current through inductor I_L, lags with applied voltage (E) by 90° . The power factor of R-L parallel circuit is cos ϕ where ϕ is the angle in between the total current and applied voltage.

Fig 3



R.L. PARALLEL CIRCUIT VECTOR

Assignment : A coil of resistance 15 ohms and inductance 0.05 H is connected in parallel with a non-inductive resistor of 40 ohms. Find the total current when a voltage of 200 V at 50 Hz. is applied. Give the phasor diagram.

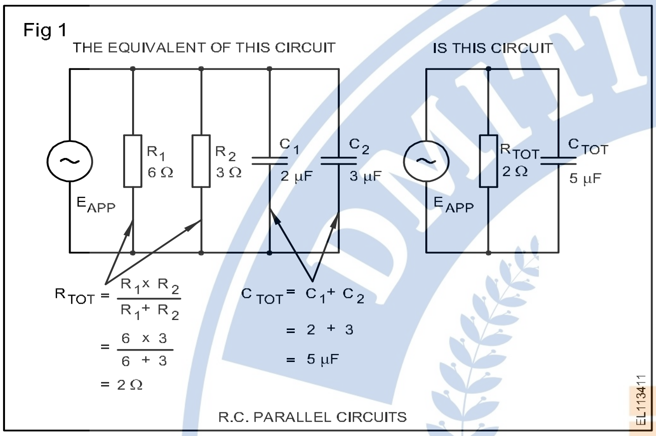
AC Parallel circuit (R and C)

Objectives: At the end of this lesson you shall be able to

- **state the relationship between branch current, voltage in a parallel circuit**
- **solve problems in RC parallel circuit by admittance method**
- **compare the charecteristics of A.C series and parallel circuits**
- **state the R-L-C parallel circuit vector diagram**

Parallel RC circuits: In a parallel RC circuit, one or more resistive loads and one or more capacitive loads are connected in parallel across a voltage source. Therefore, resistive branches, containing only resistance and capacitive branches, containing only capacitance. (Fig 1) The current that leaves the voltage source divides among the branches; so there are different currents in different branches. The current is, therefore, not a common quantity, as it is in the series RC circuits.

Fig 1



Voltage: In a parallel RC circuit, as in any other parallel circuit, the applied voltage is directly across each branch. The branch voltages are, therefore, equal to each other. So, if you know any one of the circuit voltages, you know any one of the circuit voltages, you know all of them.

Branch current: The current in each branch of a parallel RC circuit is independent of the current in the other branches. The current within a branch depends only on the voltage across the branch, and the resistance or capacitive reactance contained in it. (Fig 2)

The current in the resistive branch is calculated from the equation: $I_R = E_{APP}/R$.

The current in the capacitive branch is found with the equation: $I_C = E_{APP}/X_C$.

The current in the resistive branch is in phase with the branch voltage, while the current in the capacitive branch leads the branch voltage by 90 degrees. Since the two branch voltages are the same, the current in the capacitive branch (I_C) must lead the current in the resistive branch (I_R) by 90 degrees. (Fig 3)

Impedance: The impedance of a parallel RC circuit represents the total opposition to the current flow offered by the resistance of the resistive branch and the capacitive reactance of the capacitive branch. Like the impedance of a parallel RL circuit, it can be calculated with an equation that is similar to the one used for finding the total resistance of two parallel resistances.

Fig 2

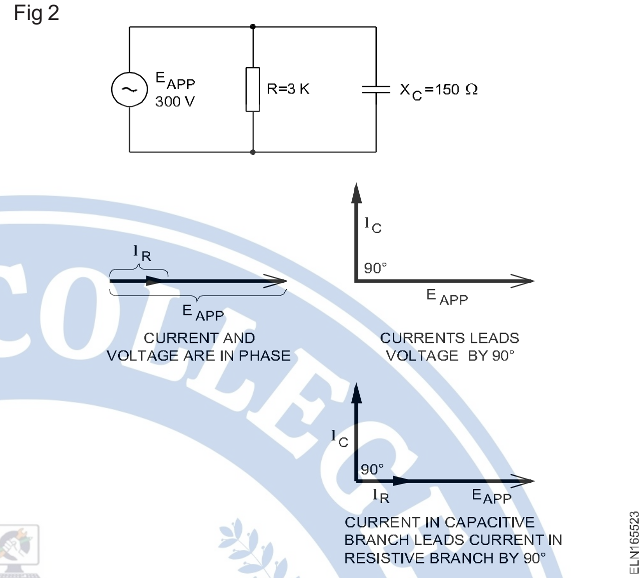
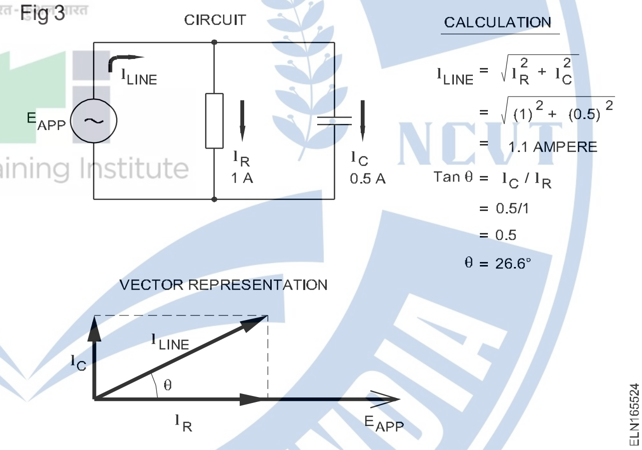


Fig 3



However, just as you learned for parallel RL circuits, two vector quantities cannot be added directly, vector addition must be used. Therfore, the equation for calculating the impedance of a parallel RC circuit is

$$Z = \frac{RX_C}{\sqrt{R^2 + X_C^2}}$$

where $\sqrt{R^2 + X_C^2}$ is the vector addition of the resistance and capacitive reactance.

In cases where you know the applied voltage and the circuit line current, the impedance can be found simply by using Ohm's law in the form:

$$Z = \frac{E_{APP}}{I_{LINE}}$$

The impedance of a parallel RC circuit is always less than the resistance or capacitive reactance of the individual branches.

The relative values of X_C and R determine how capacitive or resistive the circuit line current is. The one that is the smallest, and therefore, allows more branch current to flow, is the determining factor.

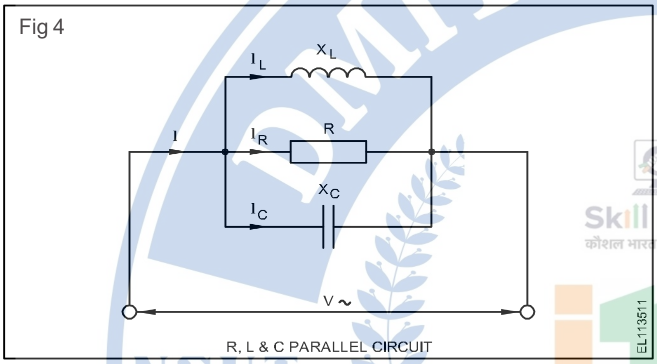
Thus, if X_C is smaller than R, the current in the capacitive branch is larger than the current in the resistive branch, and the line current tends to be more capacitive.

The opposite is true if R is smaller than X_C. When X_C or R is 10 or more times greater than the other, the circuit will operate for all practical purposes as if the branch with the larger of the two did not exist.

R, L and C Parallel circuit - Vector diagram

Parallel connection of R, X_L and X_C: X_L and X_C oppose each other, that is to say, I_L and I_C are in opposition, and partly oppose one another (Fig 4).

Fig 4



$I_X = I_C - I_L$ or $I_L - I_C$, depending on whether the capacitive or inductive current dominates.

Graphic solution: when $I_L > I_C$

1 V as common value

2 I_R in phase with V

3 I_C leads by 90°

4 I_L lags by 90°

5 $I_X = I_L - I_C$

6 I as resultant

ϕ in this case inductive, I lags (Fig 5)

Particular case: X_L and X_C are equally large - I_L and I_C cancel each other. Z = R; parallel resonance occurs.

Currents in the reactances may be greater than the total current.

The calculation of the resonant frequency is the same as for the series connection.

Example: Calculate the value of I_T, Z and power factor for the circuit in Fig 6.

Given

V_T = 10V

R = 1000 Ω

X_L = 1570 Ω

X_C = 637 Ω

Fig 5

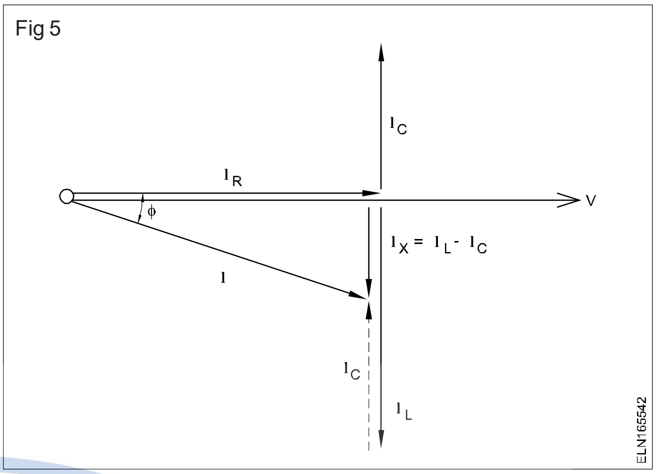
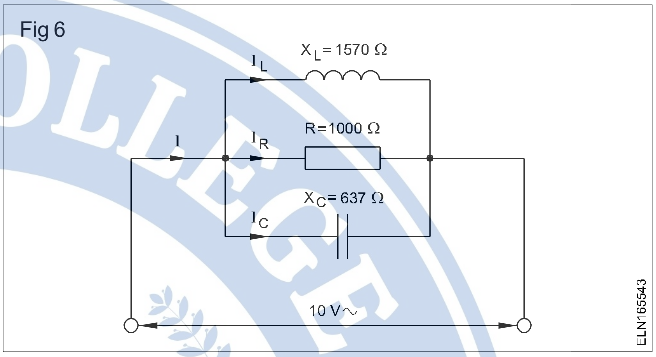


Fig 6



Known: Ohm's Law

$$I_T = \sqrt{(I_C - I_L)^2 + I_R^2}$$

Solution

$$I_C = \frac{10\ V}{637\ \Omega} = 0.0157\ A = 15.7\ mA$$

$$I_L = \frac{10\ V}{1570\ \Omega} = 0.0064\ A = 6.4\ mA$$

$$I_R = \frac{10\ V}{1000\ \Omega} = 0.01 = 10\ mA$$

$$I_T = \sqrt{(0.0157 - 0.0064)^2 + (0.01)^2}$$

$$= 0.0137 A = 13.7\ mA$$

$$Z = \frac{10V}{0.0137\ A} = 730\ \Omega$$

$$P.F = \frac{Z}{R} \quad Y = \frac{1}{Z} \quad \text{and} \quad g = \frac{1}{R}$$

$$= \frac{730}{1000} = 0.73$$

Parallel resonance circuits

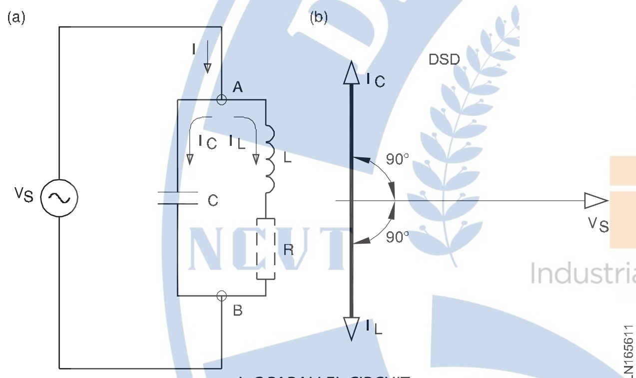
Objectives: At the end of this lesson you shall be able to
- **state the characteristics of R-L-C parallel circuits at resonance**
- **explain the term band-width in parallel LC circuits**
- **explain the storage action in parallel LC circuits**
- **list a few applications of parallel LC circuits**
- **compare the properties of series and parallel resonance circuits.**

Parallel resonance

The circuit at Fig 1, having an inductor and a capacitor connected in parallel is called parallel LC circuit or parallel resonance circuit. The resistor R, shown in dotted lines indicate the internal DC resistance of the coil L. The value of R will be so small compared to the inductive reactance, that it can be neglected.

From Fig 1a, it can be seen that the voltage across L and C is same and is equal to the input voltage V_S.

Fig 1



L-C PARALLEL CIRCUIT

By Kirchhoff's law, at junction A,

$$I = I_L + I_C.$$

The current through the inductance I_L (neglecting resistance R), lags V_S by 90°. The current through the capacitor I_C, leads the voltage V_S by 90°. Thus, as can be seen from the phasor diagram at Fig 1b, the two currents are out of phase with each other. Depending on their magnitudes, they cancel each other either completely or partially.

If $X_C < X_L$, then $I_C > I_L$, and the circuit acts capacitively.

If $X_L < X_C$, then $I_L > I_C$, and the circuit acts inductively.

If $X_L = X_C$, then $I_L = I_C$, and hence, the circuit acts as a purely resistive.

Zero current in the circuit means that the impedance of the parallel LC is infinite. This condition at which, for a particular frequency, f_r, the value of $X_C = X_L$, the parallel LC circuit is said to be in parallel resonance.

Summarizing, for a parallel resonant circuit, at resonance,

$$X_L = X_C,$$

$$Z_p = \infty$$

$$I_L = I_C$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$I = \frac{V}{Z_P} \approx 0$$

In a parallel resonance circuit, with a pure L(no resistance) and a pure C(loss-less), at resonance the impedance will be infinite. In practical circuits, however small, the inductor will have some resistance. Because of this, at resonance, the phasor sum of the branch currents will not be zero but will have a small value I.

This small current I will be in phase with the applied voltage and the impedance of the circuit will be very high although not infinite.

Summarizing, the three main characteristics of parallel resonance circuit at resonance are,

– phase difference between the circuit current and the applied voltage is zero

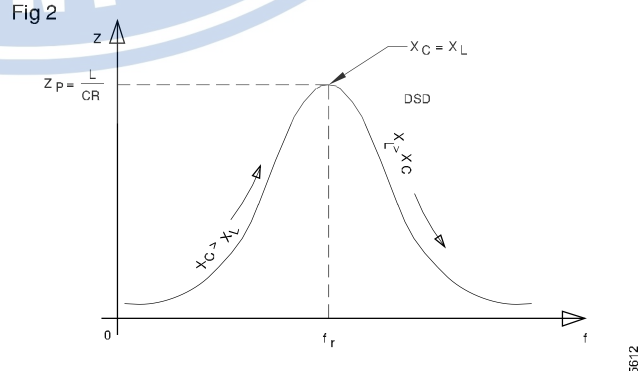
– maximum impedance

– minimum line current.

The variation of impedance of a parallel resonance circuit with frequency is shown in Fig 2.

In Fig 2, when the input signal frequency to the parallel resonance circuit is moved away from resonant frequency f_r, the impedance of the circuit decreases. At resonance the impedance Z_p is given by,

Fig 2



FREQUENCY VERSES IMPEDENCE OF A PARALLEL RESONANT CIRCUIT

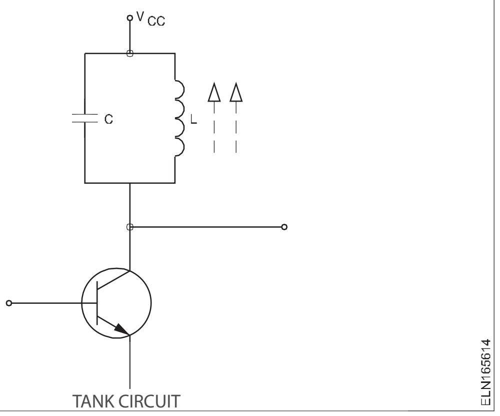
$$Z_P = \frac{L}{CR}$$

At resonance, although the circuit current is minimum, the magnitudes of I_L & I_C will be much greater than the line current. Hence, a parallel resonance circuit is also called current magnification circuit.

Application of parallel resonant circuits

Parallel resonance circuits or tank circuits are commonly used in almost all high frequency circuits. Tank circuits are used as collector load in class-C amplifiers instead of a resistor load as shown in Fig 3.

Fig 3



TANK CIRCUIT

Table below gives a comparison between series resonant and parallel resonant circuit at frequencies above and below their resonant frequency f_r.

Property	Series circuit	Parallel circuit
	At resonant frequency	
Resonant frequency, f_r	$= \frac{1}{2\pi\sqrt{LC}}$	$= \frac{1}{2\pi\sqrt{LC}}$
Reactance	$X_L = X_C$	$X_L = X_C$
Impedance	Minimum ($Z_r = R$)	Maximum ($Z_r = L/CR$)
Current	Maximum	Minimum
Quality factor	$\frac{X_L}{R}$	$\frac{X_L}{R}$
Bandwidth	$\frac{X_L}{R}$	$\frac{X_L}{R}$
	Above resonant frequency	
Reactance	$X_L > X_C$	$X_C > X_L$
Impedance	Increases	Decreases
Phase difference	The current lags behind the applied voltage.	The current leads the applied voltage.
Type of reactance	Inductive	Capacitive
	Below resonant frequency	
Reactance	$X_C > X_L$	$X_L > X_C$
Impedance	Increases	Decreases
Phase difference	The current leads the applied voltage.	The current lags behind the applied voltage.
Type of reactance	Capacitive	Inductive

Power, energy and power factor in AC single phase system - Problems

Objectives: At the end of this lesson you shall be able to
- **state the relationship between power and power factor in single phase circuits**
- **state the connection diagram for measuring power factor using a direct reading meter**
- **calculate the problem related to P.F and power in A.C circuits.**

Scan the QR Code to view the video for this exercise

The power in a DC circuit can be calculated by using the formulae.

- $P = E \times I$ watts
- $P = E^2/R$ watts.

The use of the above formulae in AC circuits will give true power only if the circuit contains pure resistance. Note that the effect of reactance is present in AC circuits.

Power in AC circuit: There are three types of power in AC circuits.

- Active power (True power)
- Reactive power
- Apparent power

Active power (True power)**:** The calculation of active power in an AC circuit differs from that in a direct current circuit. The active power to be measured is the product of $V \times I \times \cos\theta$ where $\cos\theta$ is the power factor (cosine of the phase angle between current and voltage). This indicates that with a load which is not purely resistive and where the current and voltage are not in phase, only that part of the current which is in phase with the voltage will produce power. This can be measured with a wattmeter.

Reactive power (P_r): With the reactive power (wattless power)

$$P_r = V \times I \times \sin\theta$$

only that part of the current which is 90° out of phase (90° phase shift) with the voltage is used in this case. Capacitors and inductors, on the other hand, alternatively store energy and return it to the source. Such transferred power is called reactive power measured in volt/ampere reactive or vars. Unlike true power, reacitve power can do no useful work.

Apparent power: The apparent power, $P_a = V \times I$.

The measurement can be made in the same way as for direct current with a voltmeter and ammeter.

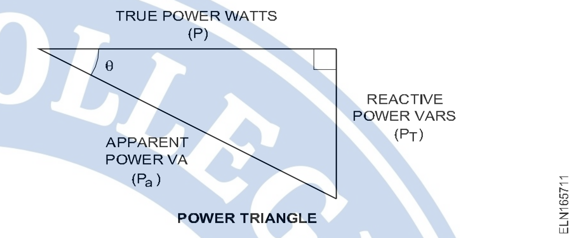
It is simply the product of the total applied voltage and the total circuit current and its unit is volt-ampere (VA).

The power triangle: A power triangle identifies three different types of power in AC circuits.

- True power in watts (P)
- Reactive power in vars (P_r)
- Apparent power VA (P_a)

The relationship among the three types of power can be obtained by referring to the power triangle. (Fig 1)

Fig 1



POWER TRIANGLE

Therefore

$P_a^2 = P^2 + P_r^2$ volt-amperes (VA)

where P_a is the apparent power in volt-ampere (VA)

P is the true power in watts (W)

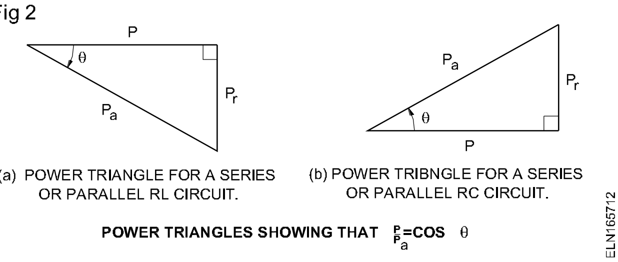
P_q is the reactive power in volt-amperes reactive. (VAR)

Power factor: The ratio of the true power delivered to an AC circuit compared to the apparent power that the source must supply is called the power factor of the load. If we examine any power triangle (Fig 2), you may see the ratio of the true power to the apparent power is the cosine of the angle θ.

$$\text{Power factor} = \frac{P}{P_a} = \cos\theta$$

From the equation, you can observe that the three powers are related and can be represented in a right-angled power triangle, from which the power factor can be obtained as the ratio of true power to apparent power. For inductive loads, the power factor is called lagging to distinguish it from the leading power factor in a capacitive load. (Fig 2)

Fig 2



(a) POWER TRIANGLE FOR A SERIES OR PARALLEL RL CIRCUIT.
(b) POWER TRIBNGLE FOR A SERIES OR PARALLEL RC CIRCUIT.

POWER TRIANGLES SHOWING THAT $\frac{P}{P_a} = \cos\theta$

A circuit's power factor determines how much current is necessary from the source to deliver a given true power. A circuit with a low power factor requires a higher current than a unity power factor circuit.

Single phase energy

The product of true power and time is known as energy.

(ie) Energy = T.Power x time

= Voltage x current x power factor x time

= VI Cos θ x t (time is in hour)

The unit of energy is watt hour and commercial unit is represented in 'KWH' (or) unit. (Board of trade unit. B.O.T)

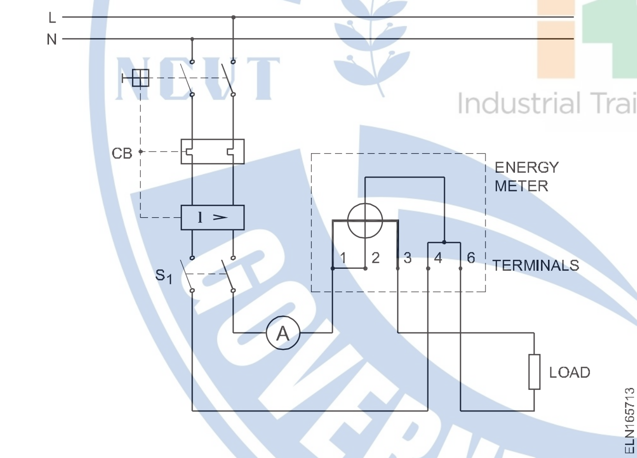
The energy depends upon the following factors:

- Voltage
- Current
- Power factor (load)
- Time

Single phase energy can be measured by energy meter. It contains 4 terminals (Incoming 2 and outgoing 2 common neutral)

The connection is shown in Fig 3.

Fig 3



AC Parallel circuit problem

In practice all industrial and domestic electrical circuits are connected in parallel as we follow the constant voltage system. In a parallel circuit, the voltage across any branch circuit is the same as the supply voltage. However, the arithmetical sum of the branch currents does not necessarily equal the total current. This is true because the branch current values may be out-of-phase due to the fact that the loads connected may be resistive, inductive, (V lead I) or capacitive (I lead V).

Therefore, the total current must be obtained by adding or subtracting vectors of the branch currents either mathematically (admittance method) or graphically (vector method).

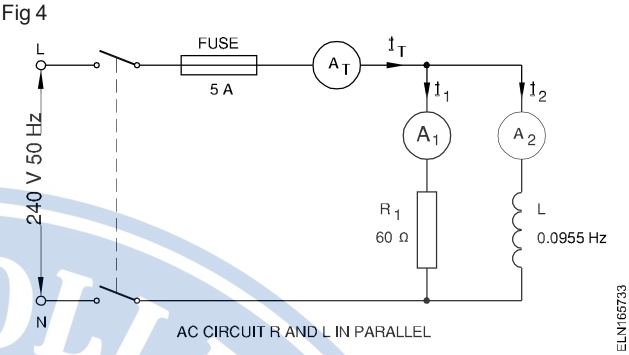
Example 1

Parallel circuit with R and X_L in branches

Now consider a parallel circuit having one branch consisting of a pure resistance and the other branch having pure inductance.

Determine the following for the circuit shown in Fig 4.

Fig 4



AC CIRCUIT R AND L IN PARALLEL

i The branch currents.

ii Draw the vector diagram.

iii The total current.

iv The power factor angle and the power factor.

v The combined impedance.

vi The power in the circuit.

SOLUTION

i The branch current $I_1 = \dfrac{V}{R_1}$

$$= \frac{240}{60} = 4 \text{ amps}$$

Pure resistive, hence, in phase with the voltage.

To calculate the branch current I_2 first find out the inductive reactance X_L.

$$X_L = 2\pi FL = 2 \times \frac{22}{7} \times 50 \times 0.0955$$

$$= 30 \text{ ohms.}$$

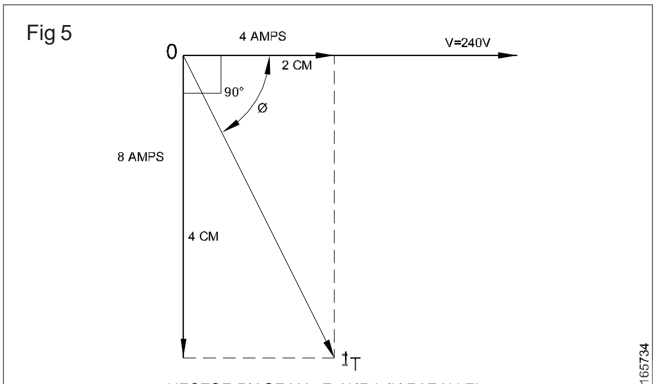
So the branch current $I_L = \dfrac{V}{X_L} = \dfrac{240}{30} = 8$ amps.

Pure inductive, hence, lags the applied voltage by 90°.

ii Draw the vector diagram by following the rules: Scale 1 cm = 2 amps. (Fig 5)

Complete the parallelogram to find the total current I_T.

Measure the angle ø and the length of $0I_T$.



VECTOR DIAGRAM - R AND L IN PARALLEL

iii Measured angle is 63° 26'

Power factor = Cos 63° 26'

= 0.447 lagging.

iv Length of $0I_T$ = 4.47 cm.

Hence, I_T = 4.47 x 2 = 8.94 amps.

The combined impedance of the circuit = Z.

v Power taken by the circuit

P = VI cos ø = I_1^2R

= 240 x 8.94 x 0.447 = 4^2 x 60

= 959 watts approx. 960 watts.

Example 2

In Fig 6, Parallel circuit with R, X_L and X_C

Find the following.

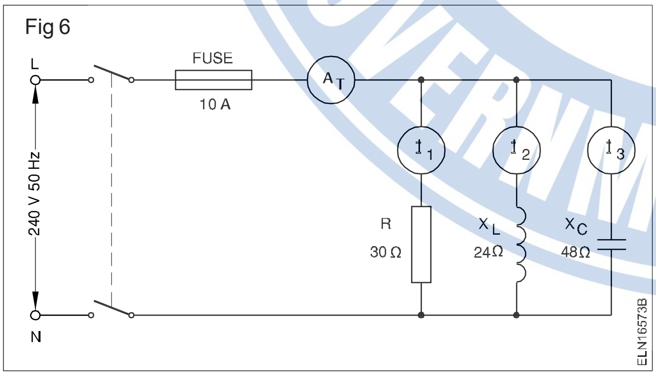
i Conductance and susceptance of each branch.

ii Total G, B and Y.

iii Branch currents.

iv PF and PF angle.

v Power taken by the circuit.



i Conductance in branch circuits

$$g_1 = \frac{R_1}{Z_1^2} = \frac{30}{30^2} = \frac{1}{30}$$

= 0.0333 siemens

$$g_2 = \frac{R_2}{Z_2^2} = \frac{0}{24^2} = 0$$

$$g_3 = \frac{R_3}{Z_3^2} = \frac{0}{48^2} = 0$$

Susceptance in branch ciruicts

$$b_1 = \frac{X_1}{Z_1^2} = \frac{0}{30^2} = 0$$

$$b_2 = \frac{X_2}{Z_2^2} = \frac{24}{24^2} = \frac{1}{24}$$

= 0.04167 siemens

$$b_3 = \frac{-X_3}{Z_1^2} = \frac{-48}{-48^2} = -\frac{1}{48}$$

= - 0.02083 siemens

ii Total conductance G = $g_1 + g_2 + g_3$

= 0.0333 + 0 + 0

= 0.0333 Siemens.

Total susceptance B = $b_1 + b_2 + b_3$

= 0 + 0.04167 + (– 0.02083)

= 0.02084 Siemens.

$$Y = \sqrt{G^2 + B^2}$$

$$= \sqrt{0.333^2 + 0.02084^2}$$

= 0.03928 Siemens.

iii The branch current $I_1 = \frac{V}{Z_1}$

$$= \frac{V}{R} = \frac{240}{30} = 8 \text{ amps in phase with V}$$

The branch current $I_2 = \frac{V}{Z_2}$

$$\frac{V}{X_L} = \frac{240}{24} = 10 \text{ amps lagging 90° with V}$$

The branch current $I_3 = \frac{V}{X_3}$

$= \frac{240}{48}$ = 5 amps lagging 90° with V

Total current

$$I_T = \sqrt{I_1^2 + (I_2 - I_3)^2}$$

$$= \sqrt{8^2 + (10-5)^2} = \sqrt{89}$$

= 9.43 amps

Alternatively

I_T = VY = 240 X 0.03928

= 9.43 amps.

iv Power factor $= \frac{G}{Y} = \frac{I_R}{I_T}$

$$= \frac{0.0333}{0.03929} = \frac{8}{9.43}$$

= 0.848.

v Power factor angle = 32° lagging.

Power taken by the circuit = VIcos ø

= 240 x 9.43 x 0.848

= 1919 watts.

Total impedance = Z = $\frac{1}{Y}$

$\frac{1}{0.03929}$ = 25.5.ohms

Check these answers with the answers obtained by the vector method.

Power factor - improvement of power factor

Objectives: At the end of this lesson you shall be able to
- **define power factor - explain the causes of low power factor**
- **list out disadvantage of low power factor and advantage of higher power factor in a circuit**
- **explain the methods to improve the power factor in an AC circuit**
- **illustrate the importance of power factor improvement in industries**
- **distinguish between leading, lagging and zero PF**
- **state the recommended power factor as per ISI 7752 (Part I) 1975 for electrical equipment.**

Power Factor (P.F.)

The power factor is defined as the ratio of true power to apparent power and it is denoted by Cos θ.

$$\text{i. e. Power Factor} = \frac{\text{True Power } (W_T)}{\text{Apparent Power} (W_a)} = \cos\theta$$

$$\text{or } \cos\theta = \frac{W_T}{V \times I}$$

Where W_T is the real power (true power) and is measured in watts or some times in kilowatts (kW). Similarly the product VI is known as the apparent power measured in volt amperes or sometimes in kilo-volt amperes written as kVA.

The principal cause of a low power factor is due to the reactive power flowing in the circuit. The reactive power mostly due to inductive load rather than capacitive load.

Variation in power factor and the type of circuits

The following are the different conditions of the power factor in different circuits.

Unity power factor

A circuit with a unity power factor will have equal real and apparent power, so that the current remains in phase with the voltage, and hence, some useful work can be done. (Fig 1a)

Leading power factor

A circuit will have a leading power factor if the current leads voltage by an angle of Ø electrical degrees and the true power will be less than the apparent power. Mostly capacitive circuits and synchronous motors operated at over excitation contribute for leading power factor. (Fig 1b)

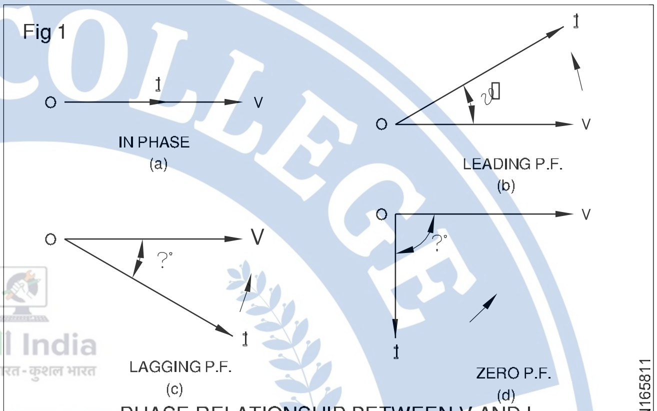
Lagging power factor

In such a circuit the true power is less than the apparent power and current lags behind the voltage by an angle, in electrical degrees. Mostly inductive loads like induction motors and induction furnaces account for lagging power factor. (Fig 1c)

Zero power factor

When there is a phase difference of 90° between the current and voltage, the circuit will have zero power factor and no useful work can be done. Pure inductive or pure capacitive circuits account for zero power factor. (Fig 1d)

Fig 1



PHASE RELATIONSHIP BETWEEN V AND I

> **The power factor can be one or less than one but can never be greater than one.**

Table 1 shows the most common electrical appliances used, the power in watts and the average power factor.

Causes of low power factor

The following are the reasons.

i In industrial and domestic fields, the induction motors are widely used. The induction motors always take lagging current which results in low power factor.

ii The industrial induction furnaces have low power factor.

iii The transformers at substations have lagging power factor because of inductive load and magnetising currents.

iv Inductive load in houses like fluorescent tubes, mixers, fans etc.

The disadvantages of low power factor are as follows.

a For a given true power, a low power factor causes increased current, thereby, overloading of the cables, generators, transmission and distribution lines and transformers.

b Decreased line voltage at the point of application (voltage drop at consumer end) due to voltage drop and power losses in the supply system.

c Penal power rates (increased electricity bills).

The advantages of high power factor are as follows.

As the higher PF for a given load, reduces the current, there will be:

a a possibility of connecting extra load on existing generators and transmit additional power through the same lines

b lesser losses and voltage drop in lines; thereby, transmission efficiency is high and the voltage at the point of application will be normal without much drop

c normal voltage improves the efficiency of operation of plants and machinery

d reduction in electricity bills for the given load during the given time.

Method of improving the power factor

To improve the power factor of a circuit, two methods are used:

i to run a lightly loaded synchronous motor with over-excitation on that line in which the PF is to be improved

ii to connect capacitors in parallel with the load.

Usually the capacitor method is used in Indian factories.

Synchronous condenser method

The synchronous motor is used in certain industries as well as in receiving end substations to drive a mechanical load and also to correct the power factor. An over-excited synchronous motor draws leading current to compensate the lagging current taken by the other loads.

The leading volt-ampere reactive power taken by a synchronous motor, when over- excited will be opposite in nature to the lagging voltage pure reactive due to inductive loads, and, thereby, reduces the volt-ampere reactive component to improve the power factor.

Condenser method

Capacitors when used for PF improvement are connected in parallel to the supply. In three-phase circuits the capacitors are connected in delta across the load lines. Now automatic devices are available which can be connected to the supply lines to detect low power factor and to switch on the required capacity of capacitors in the line to improve the power factor.

Normally these capacitors are provided with discharge resistances to discharge the stored energy. However, no capacitor terminal should be touched to avoid shock.

TABLE 1

Power factor for single phase electrical appliances and equipment (Reference IS 7752 (Part I) - 1975)

Sl.No.	Appliance/Equipment	Power output		Average natural power factor
		Min.(W)	Max.(W)	
1	Neon sign	500	5000	0.5 to 0.55
2	Window type air- conditioners	750	2000*	0.75 to 0.85
				0.68 to 0.82
				0.62 to 0.65
3	Mixer	150	450	0.8
4	Coffee grinder	200	400	0.75
5	Refrigerator	200	800	0.65
6	Freezer	600	1000	0.7

Assignment

A factory is having a load of 100 kW working at 0.6 PF lagging. A synchronous motor is connected in the factory and is made to run over-excited to improve the power factor. The synchronous motor is of 30 kW and is working at 0.8 PF leading. Calculate the following:

i the true power in watt, aspperent power in VAR for the factory load at 0.6p.f lagging.

ii The true power in watt, apparent power in volt- ampere and leading reactive power in VAR for the synchoronous motor at 0.8P.F lagging.

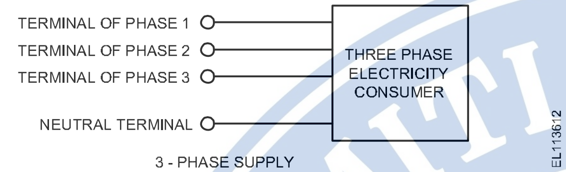
iii The true power in watt, reactive power in VAR and apparent power in Volt - ampere and PF supplied by the feeder lines.

3-Phase AC fundamentals

Objectives: At the end of this lesson you shall be able to
- **state and describe the generation of 3-phase system with single loops**
- **state the advantages of the 3-phase system over a single phase system**
- **state and explain the 3-phase, 3-wire, and 4-wire system**
- **state and explain the relation between phase and line voltage.**

A three-phase power consumer is provided with the terminals of three phases. (Fig 1)

Fig 1



One great advantage of a three-phase AC supply is that it can produce a rotating magnetic field when a set of stationary three-phase coils is energized from the supply. This is the basic operating principle for most modern rotating machines and, in particular, the three-phase induction motor.

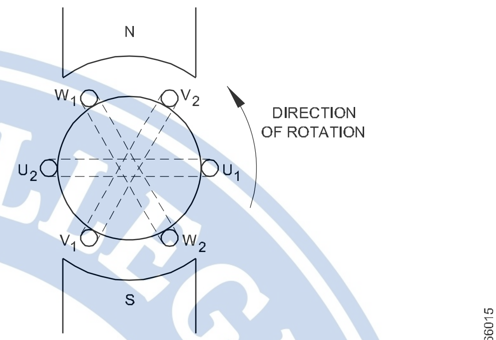
Further, lighting loads can be connected between any one of the three phases and neutral.

Review: Further to the above two advantages the following are the advantages of polyphase system over single phase system.

- 3-phase motors develop uniform torque whereas single phase motors produce pulsating torque only
- Most of the 3-phase motors are self starting whereas single phase motors are not
- Power factor of 3-phase motors are reasonably high when compared to single phase motors
- For a given size the power out put is high in 3-phase motors whereas in single phase motors the power output is low.
- Copper required for 3-phase transmission for a given power and distance is low when compared to single phase system.
- 3-phase motor like squirrel cage induction motor is robust in construction and more are less maintenance free.

Three-phase generation: To generate three-phase voltages, a similar method to that used for generating single-phase voltages is employed but with the difference that, this time, three wire loops U_1, U_2, V_1, V_2 and W_1, W_2 rotate at a constant angular speed about the same axis in the uniform magnetic field. U_1, U_2, V_1, V_2 and W_1, W_2, are displaced 120° in position with respect to each other, permanently. (Fig 2)

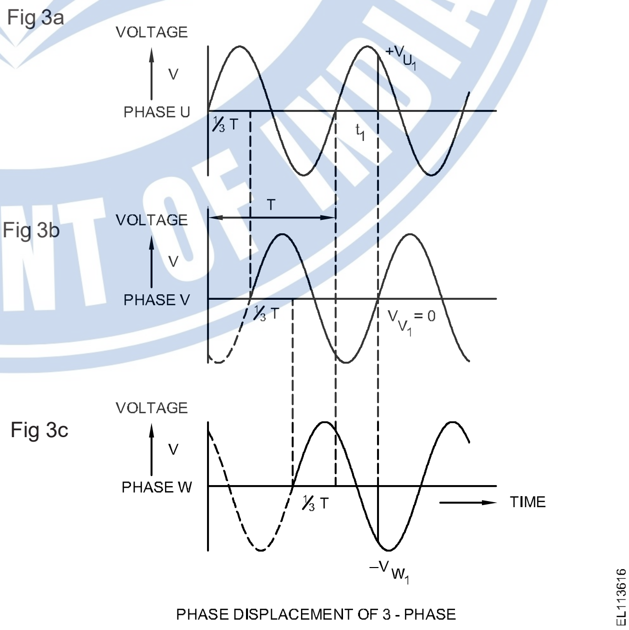
Fig 2



SIMPLIFIED THREE – PHASE ALTERNATOR

For each wire loop, the same result is obtained as for the alternating voltage generator. This means that an alternating voltage is induced in each wire loop. However, since the wire loops are displaced by 120° from each other, and a complete revolution (360°), takes one period, the three induced alternating voltages are delayed in time by a third of a period with respect to each other.

Because of the spatial displacement of the three wire loops by 120°, three alternating phase voltages result, which are displaced by one third of a period, T, with respect to each other. (Fig 3)



PHASE DISPLACEMENT OF 3 - PHASE

To distinguish between the three phases, it is a common practice in (heavy current) electrical engineering to designate them by the capital letters U,V and W or by a colour code red, yellow and blue. At a time 0, U is passing through zero volts with positively increasing voltage. (Fig 3a) V follows with its zero crossing 1/3 of the period later (Fig 3b), and the same applies to W with respect to V. (Fig 3c)

In three-phase networks, the following statements can be made about the three-phase voltages.

- The three-phase voltages have the same frequency.
- The three-phase voltages have the same peak value.
- The three-phase voltages are displaced by one third of a period in time with respect to each other.
- At every instant in time, the instantaneous sum of the three voltages

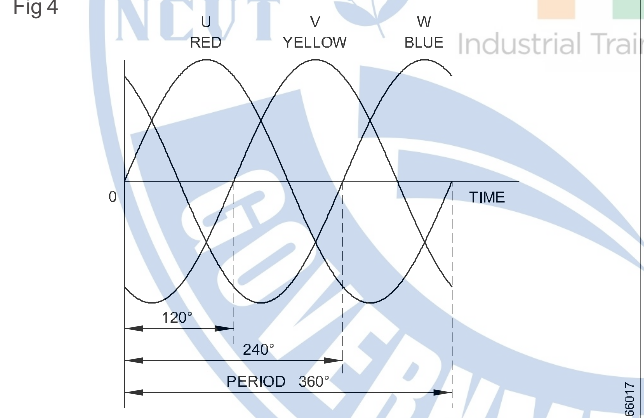
$$V_U + V_V + V_W = 0.$$

The fact that the sum of the instantaneous voltages is zero. At time t_1, U has the instantaneous value V_U. At the same time, $V_V = 0$, and the instantaneous value for W is $-V_W$. Because V_U and V_W have the same value but are opposite in sign, it follows that

$$V_{U1} + V_{V1} + V_{W1} = 0.$$

The three voltages of the same amplitude and frequency are shown together in Fig 4.

Fig 4



COMBINED WAVE - FORMS OF THREE PHASE SUPPLY

Three-phase network: A three-phase network consists of three lines or phases. In Fig 5, these are indicated by the capital letters U, V and W.

The return lead of the individual phases consists of a common neutral conductor N, which is described later in more detail. Voltmeters are connected between each of the lines U, V and W, and the neutral line N. They indicate the RMS (effective) values of the voltages between each of the three phases and neutral.

These voltages are designated as phase voltages V_{UN}, V_{VN} and V_{WN}.

The individual, phase voltages all have the same magnitude. They are simply displaced from each other by one third of a period in time. (Fig 6)

The individual instantaneous, peak and RMS values are the same as for a single-phase alternating voltage.

Fig 5

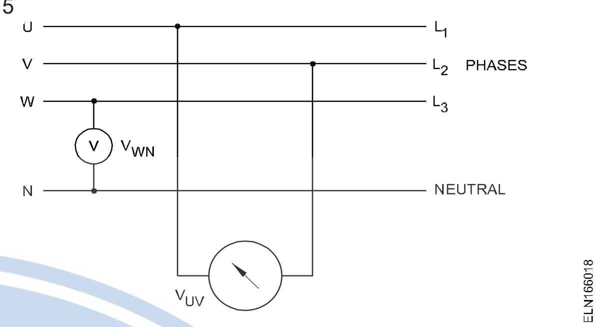
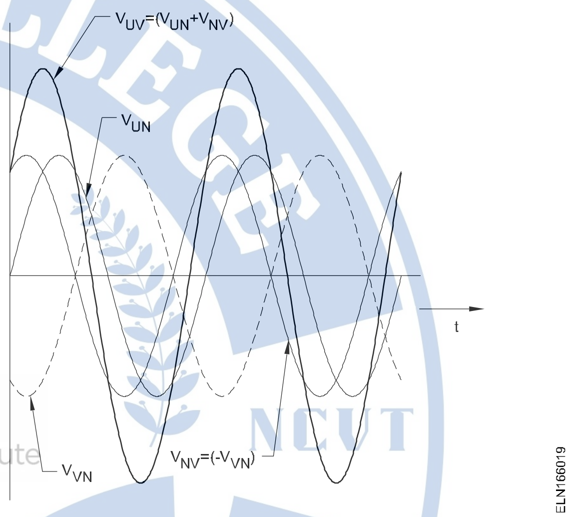


Fig 6



Line and phase voltage: If a voltmeter is connected directly between line U and line V (Fig 7), the RMS value of the voltage V_{UV} is measured, and this is different from any of the three phase voltages.

Its magnitude is directly proportional to the phase voltage. The relationship is shown in Fig 6, where the time-variation wave- forms of V_{UV} and the phase voltages V_{UN} and V_{VN} are drawn.

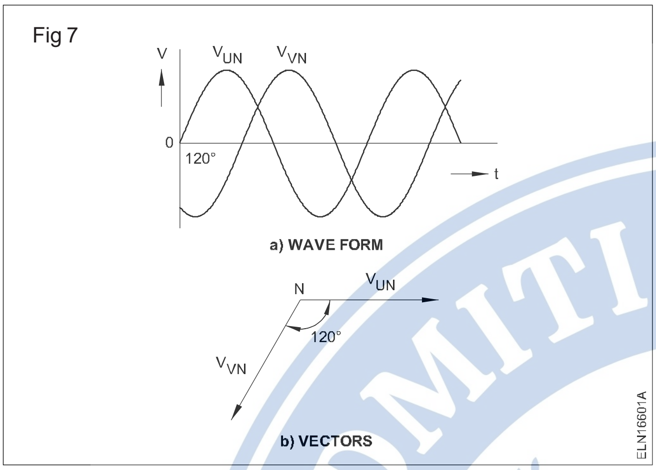
V_{UV} has a sinusoidal wave-form and the same frequency as the phase voltages. However, V_{uv} has a higher peak value since it is computed from the phase voltages V_{UN} and V_{VN}. The varying positive and negative instantaneous values of V_{UN} and V_{VN} at a particular time produce the instantaneous value of V_{UV}. V_{UV} is the phasor sum of the two phase voltages V_{UN} and V_{NV}.

This combination of phase-displaced alternating voltages is called phasor addition.

The voltage across phase-to-phase is called the line voltage.

Relationship between line and phase voltage: The possibility of combining pairs of phases in a generator is a basic property of three-phase electricity. The understanding of this relationship will be enhanced by studying the following illustrative example which explains the concept of phase difference in a very simple way.

The phase voltages V_{UN} and V_{VN} are separated in phase by one third of a period, or 120° between the two phasors. (Fig 7)



The phasor sum of the two phase voltages V_{UN} and V_{NV} can be obtained geometrically, and the resultant phasor so obtained is the line voltage V_{UV} through the relation $V_{UV} = V_{UN} + V_{NV}$.

Note that to obtain the line voltage V_{UV} the measurement is made from the U terminal through the common point N to the V terminal, for a star connection.

This fact is illustrated in Fig 8. Starting with the phasors V_{UN} and V_{VN} (Fig 7), the phasor $-V_{VN} = V_{NV}$ is produced from the point N. The diagonal of the parallelogram with sides V_{UN} and V_{NV} is the phasor representing the resulting line voltage V_{UV}.

It can be concluded, therefore, that in a generator the line voltage V_L is related to the phase voltage V_P by a multiplying factor. This factor can be shown to be $\sqrt{3}$, so that $V_L = \sqrt{3} \times V_P$

In a three-phase generating system, the line voltage is always $\sqrt{3}$ times the phase-to-neutral voltage. The factor relating the line voltage to the phase voltage is $\sqrt{3}$.

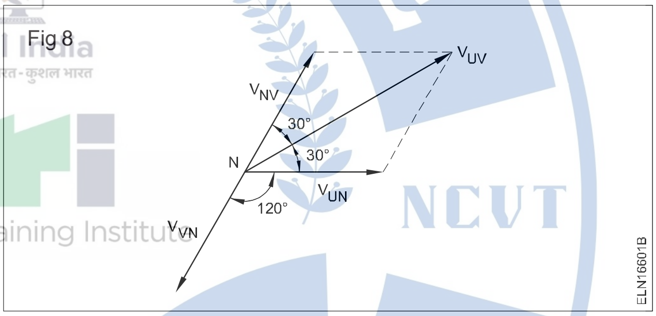
It was shown that the line voltage is greater than the phase voltage. Here is a numerical example.

The RMS phase voltage in a three-phase system is 240V. Since the ratio of line voltage to phase voltage is $\sqrt{3}$ the RMS line voltage is

$$V_L = \sqrt{3} \times V_P = \sqrt{3} \times 240$$

$$= 415.68V$$

or rounded down, $V_L = 415V$.



Systems of connection in 3-phase AC

Objectives: At the end of this lesson you shall be able to
- **explain the star and delta systems of connection**
- **state phase relationship between line and phase voltages and current in a star connection delta connection**
- **state the relationship between phase and the voltage and current in star and delta connection**

Methods of 3-phase connection: If a three-phase load is connected to a three-phase network, there are two basic possible configurations. One is `star connection' (symbol Y) and the other is `delta connection' (symbol Δ).

Star connection: In Fig 1 the three-phase load is shown as three equal magnitude resistances. From each phase, at any given time, there is a path to the terminal points U, V, W of the equipment, and then through the individual elements of the load resistance. All the elements are connected to one point N: the `star point'. This star point is connected to the neutral conductor N. The phase currents i_U, i_V, and i_W flow through the individual elements, and the same current flows through the supply lines, i.e. in a star connected system, the supply line current (I_L) = phase current (I_P).

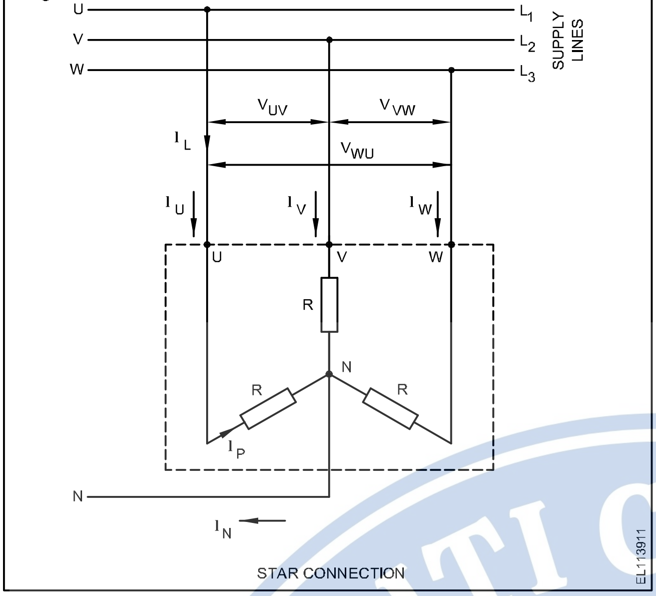
The potential difference for each phase, i.e. from a line to the star point, is called the phase voltage and designated as V_P. The potential difference across any two lines is called the line voltage V_L. Therefore, the voltage across each impedance of a star connection is the phase voltage V_P. The line voltage V_L appears across the load terminals U-V, V-W and W-U and designated as V_{UV}, V_{VW} and V_{WU} in the Fig 1. The line voltage in a star-connected system will be equal to the phasor sum of the positive value of one phase voltage and the negative value of the other phase voltage that exist across the two lines (Fig 2).

Thus

$$V_L = V_{UV} = (\text{phasor } V_{UN}) - (\text{phasor } V_{VN})$$

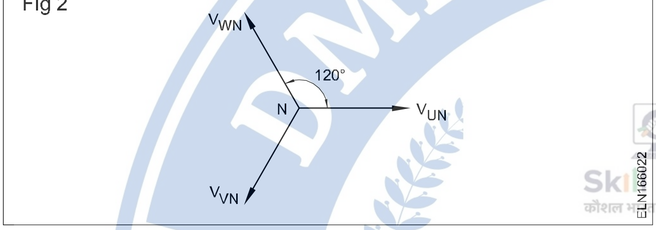
$$= \text{phasor } V_{UN} + V_{VN}.$$

Fig 1



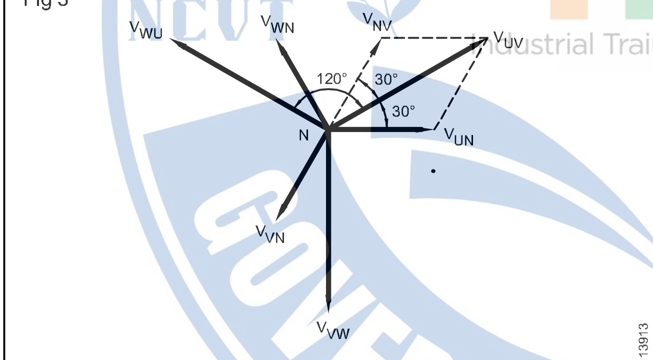
STAR CONNECTION

Fig 2



In the phasor diagram (Fig 3)

Fig 3



PHASOR DIAGRAM

$$V_L = V_{UV} = V_{UN}\ \text{Cos } 30° + V_{NV}\ \text{Cos } 30°$$

But $\text{Cos } 30° = \frac{\sqrt{3}}{2}$.

Thus as $V_{UN} = V_{VN} = V_P$

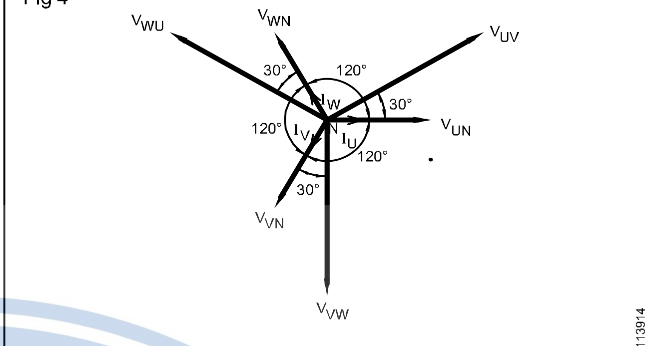
$V_L = \sqrt{3}\ V_P$.

This same relationship is applied to V_{UV}, V_{VW} and V_{WU}.

In a three-phase star connection, the line voltage is always $\sqrt{3}$ times the phase-to-neutral voltage. The factor relating the line voltage to the phase voltage is $\sqrt{3}$ (Fig 3).

The voltage and current relationship in a star connection is shown in the phasor diagrams. (Fig 4) The phase voltages are displaced 120° in phase with respect to each other.

Fig 4

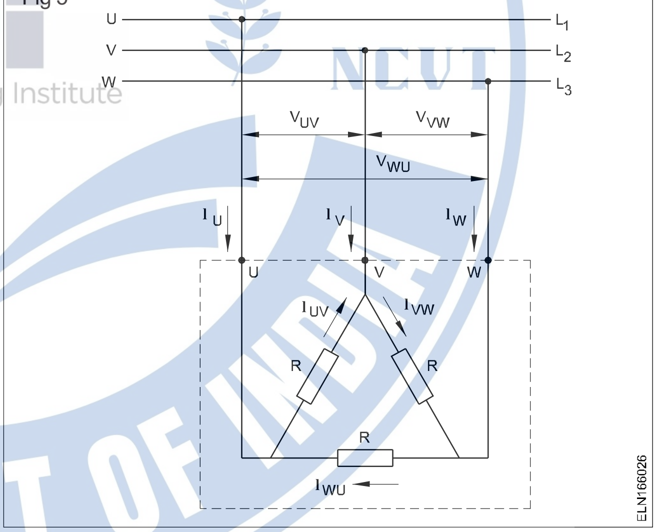


PHASOR DIAGRAM OF STAR CONNECTION

Derived from these are the corresponding line voltages. The line voltages are displaced 120° in phase with respect to each other. Since the loads in our example are provided by purely resistive impedances, the phase currents I_P (I_U, I_V, I_W) are in phase with the phase voltages V_P (V_{UN}, V_{VN} and V_{WN}). In a star connection, each phase current is determined by the ratio of the phase voltage to the load resistance R.

Delta connection: There is a second possible arrangement for connecting a three-phase load in a three-phase network. This is the delta or mesh connection (Δ).(Fig 5)

Fig 5



The load impedances form the sides of a triangle. The terminals U, V and W are connected to the supply lines of the L_1, L_2 and L_3.

In contrast to a star connection, in a delta connection the line voltage appears across each of the load phases.

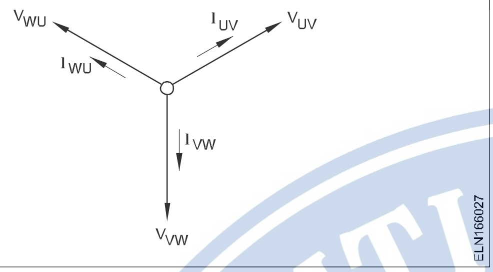
The voltages, with symbols V_{UV}, V_{VW} and V_{WU} are, therefore, the line voltages.

The phase currents through the elements in a delta arrangement are composed of I_{UV}, I_{VW} and I_{WU}. The currents from the supply lines are I_U, I_V and I_W, and one line current divides at the point of connection to produce two phase currents.

The voltage and current relationships of the delta connection can be explained with the aid of an illustration. The line voltages V_{UV}, V_{VW} and V_{WU} are directly across the load resistors, and in this case, the phase voltage is the same as the line voltage. The phasors V_{UV}, V_{VW} and V_{WU} are the line voltages. This arrangement has already been seen in relation to the delta connection.

Because of the purely resistive load, the corresponding phase currents are in phase with the line voltages. (Fig 6)

Fig 6



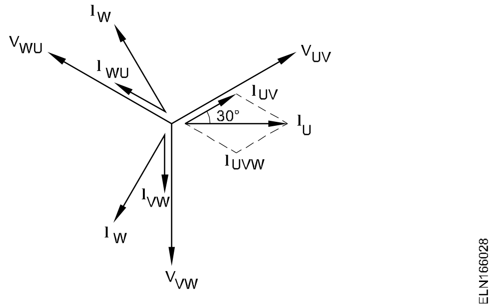
Their magnitudes are determined by the ratio of the line voltage to the resistance R.

On the other hand, the line currents I_U, I_V and I_W are now compounded from the phase currents. A line current is always given by the phasor sum of the appropriate phase currents. This is shown in Fig 7. The line current I_U is the phasor sum of the phase currents I_{UV} and I_{UW}. (See also Fig 7)

Hence, $I_U = I_{UV} \cos 30° + I_{UW} \cos 30°$

But $\cos 30° = \frac{\sqrt{3}}{2}$.

Fig 7



Thus $I_L = \sqrt{3}\, Iph$

Thus, for a balanced delta connection, the ratio of the line current to the phase current is $\sqrt{3}$.

Thus, line current = $\sqrt{3}$ x phase current.

Application of star and delta connection with balanced loads

An important application is the `star-delta change over switch' or star-delta starter.

Application of star connection: Alternators and secondoary of distribution transformers, have their three, single-phase coils interconnected in star.

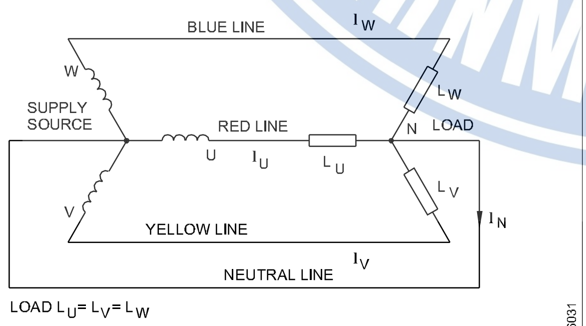
Assignment : Three identical coils, each of resistance 10 ohms and inductance 20mH is delta connected across a 400-V, 50Hz, three-phase supply. Calculate the line current.

Neutral in 3-phase system

Objectives: At the end of this lesson you shall be able to
- **explain the current in neutral of a 3-phase star connection**
- **state the earthing the neutral.**

Neutral: In a three-phase star connection, the star point is known as neutral point, and the conductor connected to the neutral point is referred as neutral conductor (Fig 1).

Fig 1



THREE-PHASE SUPPLY AND LOAD

Current in the neutral conductor: In a star-connected, four-wire system, the neutral conductor N must carry the sum of the currents I_U, I_V and I_W. One may, therefore, get the impression that the conductor must have sufficient area to carry a particularly high current. However, this is not the case, because this conductor is required to carry only the phasor sum of the three currents.

I_N = phasor sum of I_U, I_V and I_W

Fig 2 shows this phasor addition for a situation where the loads are balanced and the currents are equal. The result is that the current in the neutral line I_N is zero.

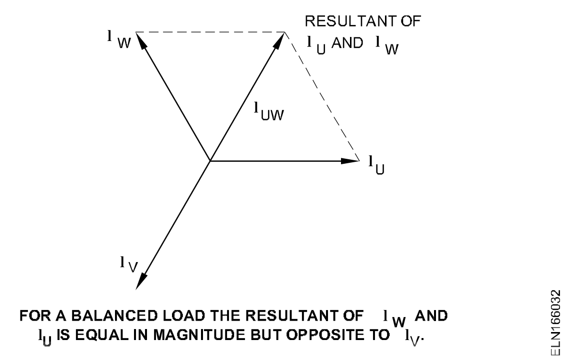
Therefore, for a balanced load the neutral conductor carries no current.

Earthing of neutral conductor: Supply of electrical energy to commercial and domestic consumers is an important application of three-phase electricity. For `low voltage distribution' - in the simplest case, i.e. supply of light and power to buildings - there are two requirements.

1 It is desirable to use conductors operating at the highest possible voltage but with low current in order to save on expensive conductor material.

2 For safety reasons, the voltage between the conductor and earth must not exceed 250V.

Fig 2



A voltage distribution system according to criterion 2, only possible with a low line voltage below 250 V. However, this is contrary to criterion 1. On the other hand, with a star connection, a line voltage of 415V is available. In this case, there is only 240V between the supply line and the neutral conductor. Criterion 1 is satisfied and, to comply with 2, the neutral conductor is earthed.

Indian Electricity Rules: I.E.Rules insist that the neutral conductor must be earthed by two separate and distinct connections to earth. Rule No.61(1)(a), Rule No.67(1)(a) and Rule No.32 insist on the identification of neutral at the point of commencement of supply at the consumer's premises, and also prevent the use of cut outs or links in the neutral conductor. BIS stipulate the method of earthing the neutral. (Code No.17.4 of IS 3043-1966)

Cross-sectional area of neutral conductor: The neutral conductor in a 3-phase, 4-wire system should have a smaller cross-section. (half of the cross-section of the supply lines).

Power in star and delta connections

Objectives: At the end of this lesson you shall be able to
- **explain active, apppparent and reactive power in AC 3 phase ϕ**
- **explain behaviour of unbalanced and balance load**
- **state the earthing the neutral.**
- **determine the power in 3-phase star and delta connected balanced load.**

Fig 1 shows the load of three resistances in a star connection. So the power must be three times as great as the single phase power.

$$P = 3V_pI_p.$$

If the quantities V_P and I_P in the individual phases are replaced by the corresponding line quantities V_L and I_L respectively, we obtain:

$$P = 3\frac{V_L}{\sqrt{3}}I_L.$$

(Because $V_P = V_L$ ¸ $\sqrt{3}$ and $I_p = I_L$)

Since $3 = \sqrt{3} \times \sqrt{3}$, this equation can be simplified to the form

$$P = \sqrt{3}\, V_L I_L$$

> **Note that power factor in resistance circuit is unity. Hence power factor is not taken into account.**

The power in this purely resistive load(ϕ=0°, cosϕ= 1) is entirely active power which is converted into heat. The unit of active power is the watt (W).

As the last formula shows, three-phase power in a star-connected load circuit can be calculated from the line quantities, and there is no need to measure the phase quantities.

$P = \sqrt{3} \times V \times I$ (Formula holds good for pure resistive load)

It is always possible, in practice, to measure the line quantities but the accessibility of the star point cannot always be guaranteed, and so it is not always possible to measure the phase voltages.

Three-phase power with a delta-connected load: Fig 2 shows the load of three resistances connected in delta. Three times the phase power will be dissipated.

Fig 1

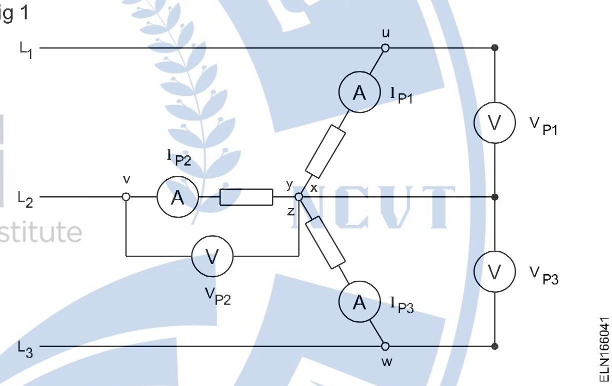
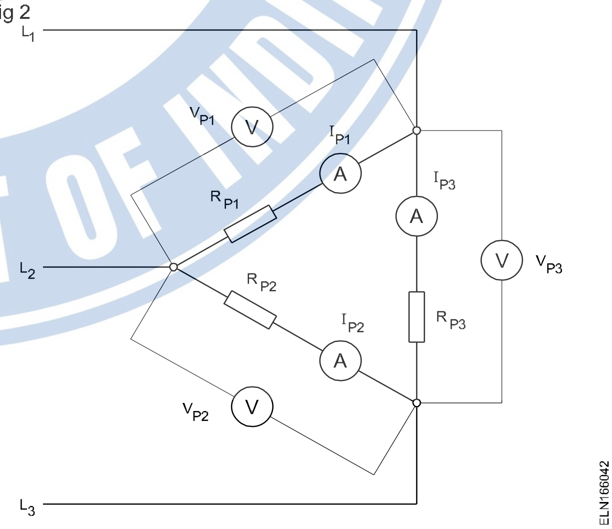


Fig 2



$$P = 3P_P = 3V_PI_P$$

If the quantities V_P and I_P are replaced by the corresponding line quantities V_L and I_L, we obtain:

Since, $V_L = V_P$

$$I_L = \sqrt{3}I_P \text{ and } I_P = \frac{I_L}{\sqrt{3}}$$

but since $3 = \sqrt{3} \times \sqrt{3}$, this equation can be simplified to the form:

$P = \sqrt{3}\ V_L I_L$.(Formula holds good for pure resistive load)

If we compare the two power formulae for the star and delta connections, we see that the same formula applies to both. In other words, the way in which the load is connected has no effect on the formula to be used, assuming that the load is balanced.

Active,reactive and apparent power: As you already know from AC circuit theory, load circuits which contain both resistance and inductance, or both resistance and capacitance, take both active and reactive power because of the phase difference existing between the voltage and current in them. If these two components of power are added geometrically, we obtain the apparent power. Precisely the same happens in each phase of the three-phase systems. Here we have to consider the phase difference f between the voltage and current in each phase.

Applying the factor $\sqrt{3}$, the components of power in a three-phase system follow from the formulae derived for single-phase, AC circuits, namely:

Apparent power	$S = VI$	$S = \sqrt{3} V_L I_L$	VA
Active power	$P = VI \cos\phi$	$P = \sqrt{3}\ V_L I_L \cos Ø$	W
Reactive power	$Q = VI \sin\phi$	$Q = \sqrt{3} V_L I_L \sin Ø$	var

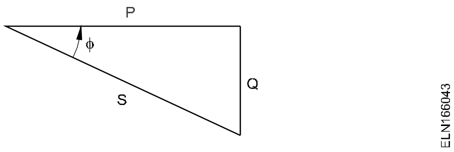
Finally, the well known relationships found in single-phase AC circuits apply also to three-phase circuits.

$$\cos\phi = \frac{\text{active power}}{\text{apparent power}} = \frac{P}{S}$$

$$\sin\phi = \frac{\text{reactive power}}{\text{apparent power}} = \frac{Q}{S}$$

This can also be seen from Fig 3.

Fig 3

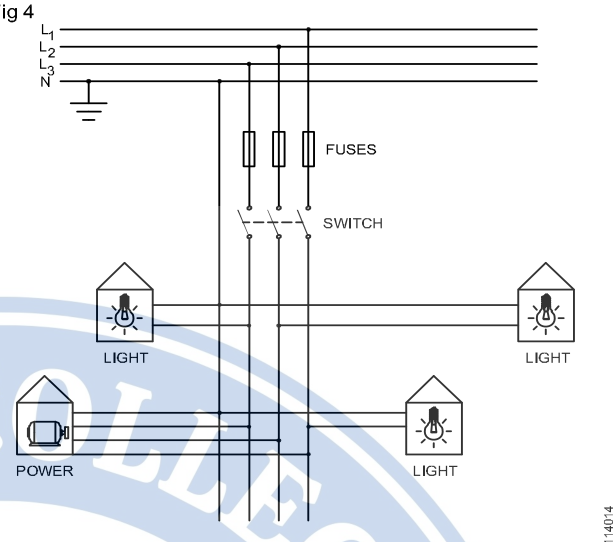


Cos ϕ is called the power factor, while sin ϕ is sometimes called the reactive power factor.

Unbalanced load: The most convenient distribution system for electrical energy supply is the 415/240 V four-wire, three-phase AC system.

This offers the possibility of supplying three-phase, as well as single-phase current, to users simultaneously. Supply to buildings can be arranged as in the given example. (Fig 4)

Fig 4



UNBALANCED LOAD

The individual houses utilize one of the phase voltages. L_1, L_2 and L_3 to N are distributed in sequence (light current). However, large loads (eg.three-phase AC motors) may be fed with the line voltage (heavy current).

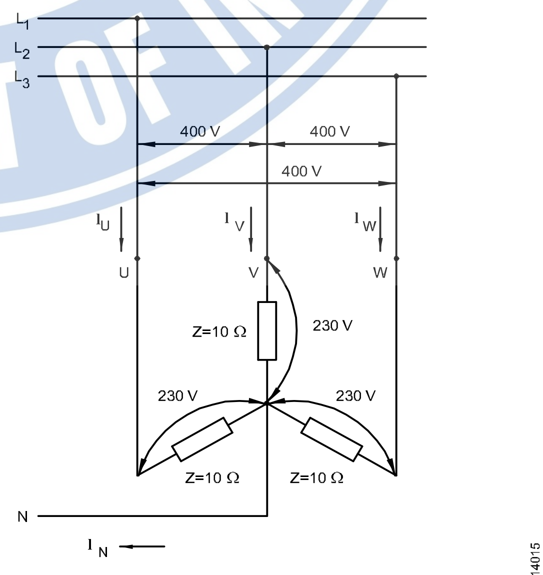
However, certain equipment which needs single or two phase supply can be connected to the individual phases so that the phases will be differently loaded, and this means that there will be unbalanced loading of the phases of the four-wire, three-phase network.

Balanced load in a star connection: In a star connection, each phase current is determined by the ratio of phase voltage and load impedance `Z'.

This fact will now be confirmed by a numerical example.

A star-connected load consisting of impedances `Z' each of 10 ohms, is connected to a three-phase network with line voltage V_L = 415V. (Fig 5)

Fig 5



BALANCED STAR CONNECTED LOAD

Because of the arrangements of a star connection, the phase voltage is 240V (415/ $\sqrt{3}$).

The three load currents taken froms supply have the same magnitude since the star-connected load is balanced, and they are given by

$$I_U = I_V = I_W = V_P \div Z.$$

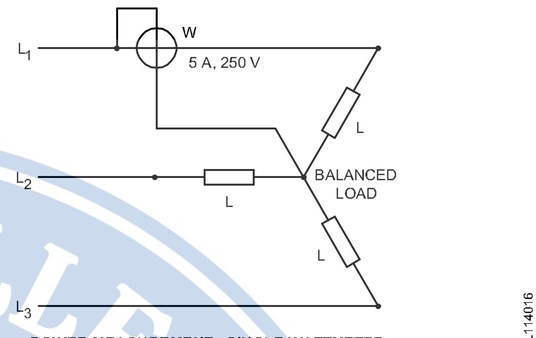
The measurement of power: The number of wattmeters used to obtain power in a three-phase system depends on whether the load is balanced or not, and whether the neutral point, if there is one, is accessible.

- Measurement of power in a a star-connected balanced load with neutral point is possible by a single wattmeter.
- Measurement of power in a star or delta-connected, balanced or unbalanced load (with or without neutral) is possible with two wattmeter method.

Single wattmeter method: Fig 6 shows the circuit diagram to measure the three-phase power of a star-connected, balanced load with the neutral point accessible the current coil of the wattmeter being connected to one line, and the voltage coil between that line and neutral point. The wattmeter reading gives the power per phase. So the total is three times the wattmeter reading.

$$\text{Power/phase} = 3V_P I_P \cos\theta = 3P = 3W.$$

Fig 6



POWER MEASUREMENT - SINGLE WATTMETER

The two-wattmeter method of measuring power

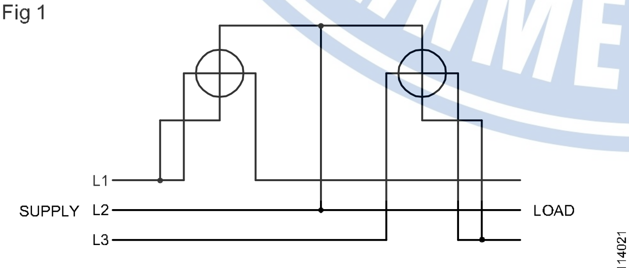
Objectives: At the end of this lesson you shall be able to:
- **measure 3-phase power using two single phase wattmeter**
- **calculate power factor from meter reading**
- **explain the `two-wattmeter' method of measuring power in a three-phase, three-wire system.**

Power in a three-phase, three-wire system is normally measured by the `two-wattmeter' method. It may be used with balanced or unbalanced loads, and separate connections to the phases are not required. This method is not, however, used in four-wire systems because current may flow in the fourth wire, if the load is unbalanced and the assumption that $I_U + I_V + I_W = 0$ will not be valid.

The two wattmeters are connected to the supply system as shown in Fig 1. The current coils of the two wattmeters are connected in two of the lines, and the voltage coils are connected from the same two lines to the third line. The total power is then obtained by adding the two readings:

$$P_T = P_1 + P_2.$$

Fig 1



TWO WATTMETER - POWER MEASUREMENT

Consider the total instantaneous power in the system $P_T = P_1 + P_2 + P_3$ where P_1, P_2 and P_3 are the instantaneous values of the power in each of the three phases.

$$P_T = V_{UN}\, i_U + V_{VN}\, i_V + V_{WN}\, I_W$$

Since there is no fourth wire, $i_U + i_V + i_W = 0$; $i_V = -(i_U + i_W)$.

$$P_T = V_{UN} i_U - V_{VN}(i_U + i_W) + V_{WN}\, i_W$$
$$= i_U(V_{UN} - V_{VN}) + i_W(V_{WN} - V_{UN})$$
$$= i_U V_{UV} + i_W V_{WV}$$

Now $i_U V_{UV}$ is the instantaneous power in the first wattmeter, and $i_W V_{WV}$ is the instantaneous power in the second wattmeter. Therefore, the total mean power is the sum of the mean powers read by the two wattmeters.

It is possible that with the wattmeters connected correctly, one of them will attempt to read a negative value because of the large phase angle between the voltage and current for that instrument. The current coil or voltage coil must then be reversed and the reading given a negative sign when combined with the other wattmeter readings to obtain the total power.

At unity power factor, the readings of two wattmeter will be equal. Total power = 2 x one wattmeter reading.

When the power factor = 0.5, one of the wattmeter's reading is zero and the other reads total power.

When the power factor is less than 0.5, one of the wattmeters will give negative indication. In order to read the wattmeter, reverse the pressure coil or current coil connection. The wattmeter will then give a positive reading but this must be taken as negative for calculating the total power.

When the power factor is zero, the readings of the two wattmeters are equal but of opposite signs.

Power factor calculation in the two-watmeter method of measuring power

As you have learnt in the previous lesson, the total power $P_T = P_1 + P_2$ in the two-wattmeter method of measuring power in a 3-phase, 3-wire system.

From the readings obtained from the two wattmeters, the tan ϕ can be calculated from the given formula

$$\tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)} = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

from which ϕ and power factor of the load may be found.

Example 1*:* Two wattmeters connected to measure the power input to a balanced three-phase circuit indicate 4.5 KW and 3 KW respectively. Find the power factor of the circuit.

Solution

$$\tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)}$$

$P_1 = 4.5$ KW

$P_2 = 3$ KW

$P_1 + P_2 = 4.5 + 3 = 7.5$ KW

$P_1 - P_2 = 4.5 - 3 = 1.5$ KW

$$\tan \phi = \frac{\sqrt{3} \times 1.5}{7.5} = \frac{\sqrt{3}}{5} = 0.3464$$

$\phi = \tan^{-1} 0.3464 = 19^0 6'$

Power factor $\cos 19^0 6' = 0.95$

Phase-sequence indicator (Meter)

Objectives: At the end of this lesson you shall be able to

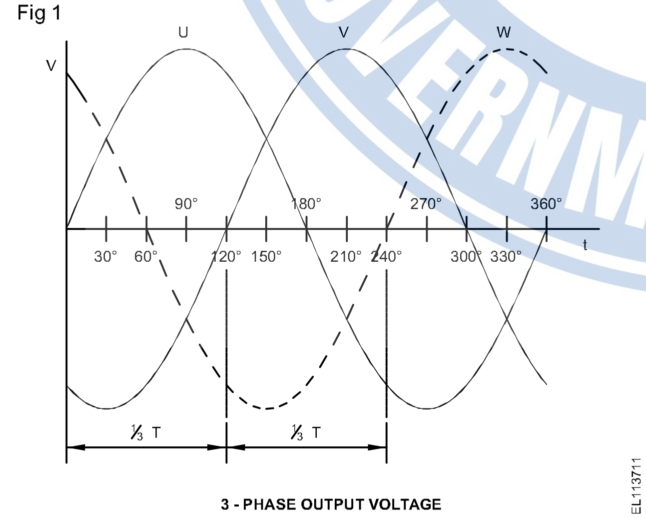
- **describe the method of finding the phase sequence of a 3-phase supply using a phase-sequence indicator**
- **explain the methods of finding phase sequence using lamps.**

Phase sequence

A three-phase alternator contains three sets of coils positioned 120° apart and its output is a three-phase voltage as shown in Fig 1. A three-phase voltage consists of three voltage waves, 120 electrical degrees apart.

At a time 0, phase U is passing through zero volts with positively increasing voltage. (Fig 1) V follows with its zero crossing 1/3 of the period later and the same applies to W with respect to V. The order in which the three-phases attain their maximum or minimum values is called the phase sequence. In the illustration given here the phase sequence is U,V,W.

Fig 1

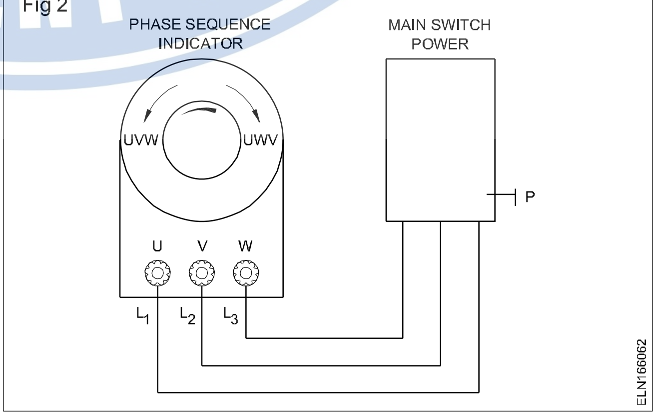


Importance of correct phase sequence: Correct phase sequence is important in the construction and connection of various three-phase systems. For example, correct phase sequence is important when the outputs of three-phase alternators must be paralleled into a common voltage system. The phase `U' of one alternator must be connected to phase `U' of another alternator. The phase `V' to phase `V' and phase `W' to phase `W' must be similarly connected to each other.

In the case of an induction motor, reversal of the sequence results in the reversal of the direction of motor rotation which will drive the machinery the wrong way.

Phase-sequence indicator(meter)**:** A phase-sequence indicator (meter) provides a means of ensuring the correct phase-sequence of a three-phase system. The phase-sequence indicator consists of 3 terminals `UVW' to which three-phases of the supply are connected. When the supply is fed to the indicator a disc in the indicator moves either in the clockwise direction or in the anticlockwise direction. The direction of the disc movement is marked with an arrowhead on the indicator. Below the arrowhead the correct sequence is marked. (Fig 2)

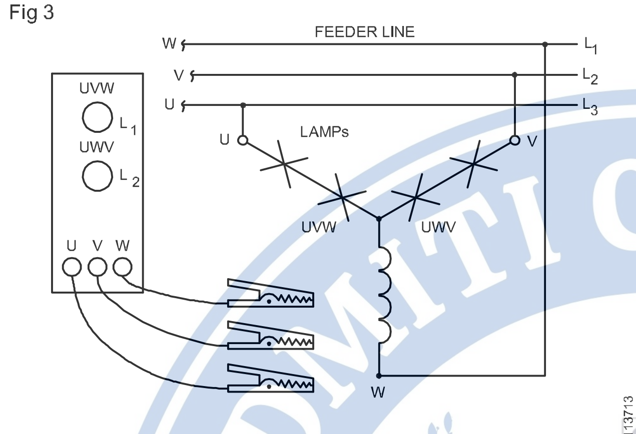
Fig 2



The phase sequence of the three-phase system may be reversed by interchanging the connections of any two of the three phases.

Phase-sequence indicator using choke and lamps: The phase-sequence indicator consists of four lamps and an inductor connected in a star formation (Y). A test lead is connected to each leg of the `Y'. One lamp is labelled U-V-W, and the other is labelled U-W-V. When the three leads are connected to a three-phase line, the brighter lamp indicates the phase sequence. (Fig 3)

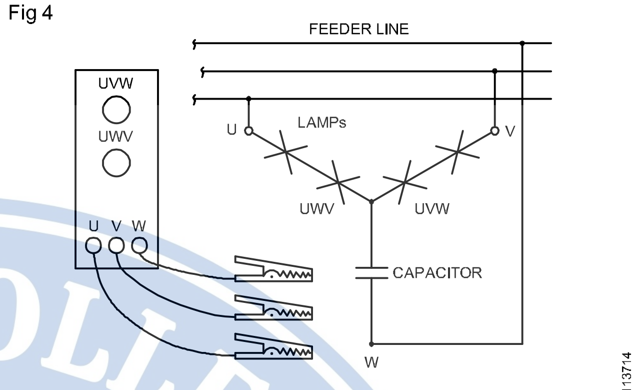
Fig 3



PHASE SEQUENCE INDICATOR USING CHOKE & LAMPS

Phase-sequence indicator using capacitor & lamps: The phase-sequence indicator consists of four lamps and a capacitor connected in a star formation (Y). A test lead is connected to each leg of the `Y'. One pair of lamps are labelled U-V-W,and the other pair are labelled U-W-V. When the three leads are connected to a 3-phase line, the brighter lamp indicates the phase sequence. (Fig 4)

Fig 4



PHASE SEQUENCE INDICATOR USING CAPACITOR & LAMPS